On Price Dispersion, Search Externalities, and the Digital Divide

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Abstract

We propose a model of price competition where consumers exogenously differ in the number of prices they compare. Our model can be interpreted either as a non–sequential search model or as a network model of price competition. We show that i) if consumers who previously just sampled one firm start to compare more prices all types of consumers will expect to pay a lower price and ii) if consumers who already sampled more than one price sample (even) more prices then there exists a threshold –the digital divide– such that all consumers comparing fewer prices than this threshold will expect to pay a higher price whereas all consumers comparing more prices will expect to pay a lower price than before. In addition, we exhibit an example where all types of consumers will expect to pay higher prices, as already informed consumers start to sample more prices.

JEL Classification: D43, D85, L11

1. Introduction

What we observe in daily life is a stark dispersion of prices at which homogenous goods are offered by different firms, i.e. we observe price dispersion. In a recent survey Baye, Morgan, and Scholten (2006) report on dozens of empirical papers providing ample evidence in both on- and off-line consumer good markets† for Varian’s (1980) assertion that actually “the law of one price is no law at all”.

The classic explanation for persistent price dispersion is provided by models where firms compete on prices and consumers differ (either exogenously or endogenously) in the number of prices they compare. Following the works of Wilde and Schwartz (1979) and Varian (1980), in many of these models there are

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1The recent rise of internet price comparison sites has not only eased consumer search but has made it also much easier for researcher to collect data, hence the second wave in the literature.
only two types of consumers.\(^2\) A fraction of consumers—the informed consumers—compare all prices and buy from the firm offering the lowest price. The remaining fraction—the uninformed consumers—just sample one price at random. The presence of these two groups of consumers implies that firms face a tradeoff between setting low prices in order to attract a high number of informed consumers and setting high prices to rip off the uninformed consumers. These two forces are balanced in a mixed strategy equilibrium where firms randomize on prices, which can be interpreted as the theoretical equivalent of the empirical observation of price dispersion.

In models of consumer search the division of consumers in uninformed consumers and informed consumers arises endogenously as consumers decide on the optimal number of prices to sample. Burdett and Judd (1983) present a model where consumers have homogeneous search costs and engage in optimal non–sequential search. In this setup consumers either find it optimal to sample only one price or randomize between sampling one and two prices. In the latter case consumers endogenously divide themselves into uninformed and (partially) informed consumers. Stahl (1989) presents a model of sequential search where a fraction of consumers has zero search costs and samples all firms in the market, and the remaining fraction has positive search costs and samples only one firm in the market. Janssen and Moraga-González (2004) analyze the case of non–sequential search in the presence of consumers with zero search costs and consumers with positive search costs. Naturally, consumers with zero search costs will always sample all firms in the market. Depending on the search cost, consumers with positive search will either randomize between not sampling (and not buying) and sampling one price (and buying), or sample one price, or randomize between sampling one price and two prices.

In the present paper we aim at exploring a model that allows for any degree of partially informed consumers. Some consumers will only be visiting one firm, some consumers will be comparing the prices of two firms, some of three firms, and so forth, obtaining a consumer search distribution. Burdett and Judd (1983) already show existence of a mixed strategy equilibrium under such a general consumer search distribution. In their model consumers with homogeneous search costs find it optimal to either visit one firm or to randomize between visiting one and two firms.\(^3\) We assume that the consumer search distribution is exogenously given and remark that any consumer search distribution can be justified as the result of consumers with sufficiently heterogenous search costs engaging in optimal non–sequential search.

The first advantage of looking at more general consumer search distributions is that we are able to provide new insights into the structure of price dispersed equilibria by varying the consumer search distribution. We find that the only determinant for total consumer welfare is the number of uninformed consumers. That is neither the number of firms\(^4\) nor the exact distribution of the informed consumers matters. In line with the previous literature on consumer search we find that lowering the fraction of uninformed consumers tightens competition and lowers expected prices for all different types of consumer. One might be tempted by this result to think that “more search” is always beneficial to all consumers. We show that this is not the case if already informed consumers start to compare more prices. In particular we show that if already informed consumers start to compare even more prices then there exists a threshold—the digital divide—such that everybody who is comparing less prices than this threshold will expect to

\(^2\)The survey by Baye, Morgan, and Scholten (2006) reports on many more theoretical papers on price dispersion and consumer search.

\(^3\)Burdett and Judd (1983) also analyze the case of noisy search, where consumers decide whether to receive an unknown number of price observations.

\(^4\)Provided that in the characterization there are sufficiently many firms for the most informed consumers to sample from.
pay a higher price whereas everybody comparing more prices will expect to pay lower prices than before. In this sense, more search by informed consumers inflicts a negative (pecuniary) search externality upon relatively poorly informed consumer whereas it causes a positive search externality for relatively well informed consumers. Figure 1 illustrates this point by the mean of an example.

Figure 1: The effect of already informed consumers engaging in even more search on expected prices of different types of consumers. The circles represent expected prices under the original consumer search distribution $q = \frac{1}{100}(47, 20, 10, 10, 1, 1, 1)$ and the rectangles represent expected prices after 10% of the consumers who previously sampled two firms sample now five firms. After this shift consumers who sample three or less than three prices will expect to pay higher prices while consumers sampling more than three prices expect to pay a lower price.

These findings are at the heart of the current political debate on broadband stimulus which turns around the question whether existing internet connections should be made faster or the internet should be made accessible to a broader audience. The Economist argues that:

... But the case for large-scale government investment in broadband is not as strong as its proponents claim. When it comes to promoting economic activity, it is easy to see why having broadband is better than not having it, but most benefits are likely to come from wiring people up in the first place rather than making existing connections hum faster... (“Broadband Stimulus,” The Economist, January 31st 2009, Vol. 380, Number 8616, p12.)

Our model supports this viewpoint, as making existing connections faster would merely result in a re-distribution of social welfare from relatively poor informed consumers to better informed consumers, whereas wiring up new consumers would increase total consumer welfare.

A further advantage of considering general consumer search distributions is that it allows us to put forward the interpretation of the search model at hand as a model of price competition on a network in
the following sense. Firms and consumers represent nodes in a bipartite network, i.e. a network where there are only links between firms and consumers. A link between a consumer and a firm in this network indicates that a consumer observes the price of this firm and may buy from this firm and that the firm may sell to this consumer. If we assume that firms only know the degree distribution of consumers, i.e. the the probabilities that a given consumer has one link, two links, and so forth, we obtain a network game of incomplete information. Alternatively, we could also assume that the network is symmetric in the sense that every firm faces the same consumer degree distribution and analyze this game of complete information. Note that the number of potential consumers a firm can attract (i.e. its own degree) may be different across firms, though.

Within this network interpretation the consumer degree distribution essentially plays the role of the consumer search distribution in the consumer search model. In this sense, the equilibrium we find in the search context and the comparative static exercise of varying the consumer search distribution translate into our network context. In particular, the comparative static exercise of introducing more search can be interpreted as increasing the density of the network. We then have that adding links to consumers who are only linked to one firm decreases expected prices for all consumers, whereas adding links to consumers with already more than one link amounts to a redistribution of social welfare from consumers who just have a few links to well connected consumers.

This second interpretation of our model is related to the literature of competition on networks. Kranton and Minehart (2001) and Corominas-Bosch (2004) analyze bargaining situations between fully informed agents on exogenously given non–regular networks. These models however become soon very complex, untractable and with multiple equilibria, as the size of the network grows. Blume, Easley, Kleinberg, and Tardos (2009) present instead a model with uniqueness of equilibria, with sellers, buyers, and traders, where traders play the role of mediators and may set different prices to different buyers and sellers. Lever (2008) studies duopolistic price competition under complete information on a network, where of course consumers may at most have degree two.

The rest of the paper is organized in the following way: Section 2 spells out the model. Section 3 presents the results and section 4 concludes.

2. The Model

We consider a market for a homogenous good with $N$ firms and $M$ households (consumers), and write $\mu = M/N$ for the number of households per firm. Each firm can produce the good at constant marginal cost, without fixed cost, and sets the price at which it offers the good (all firms set their prices simultaneously). Each household demands one unit of the good, up to a reservation price (assumed greater than the cost). Without loss of generality we normalize the cost to 0 and the reservation price to 1 (the same for all firms, respectively households).

Households differ in the information they have about the firms’ prices. A household of type $k$ observes the prices of $k$ firms and buys from the cheapest (randomizing with equal probabilities in case of ties),

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5Galeotti (2008) presents a model of search and word of mouth communication, where a network determines the communication process.
6This incomplete information setup stipulates the use of the Bayesian-Nash equilibrium concept for networks, as e.g. in Jackson and Yariv (2007) or Galeotti, Goyal, Jackson, Vega-Redondo, and Yariv (2009).
provided the price does not exceed the reservation price.

We denote by \( q_k \) the fraction of households of type \( k \). The information structure is represented by the vector \( q = (q_1, \ldots, q_N) \), where of course \( q_k \geq 0 \) for all \( k \) and \( \sum_{k=1}^{N} q_k = 1 \). We also refer to \( q \) as the consumer search distribution or the degree distribution.

As already mentioned in the Introduction, there are two interpretations for this kind of model: (1) non-sequential search: the \( k \) firms which a household of type \( k \) observes are a random sample (with equal probabilities) from the set of all \( N \) firms; and (2) a bipartite network of firms and households: a household and a firm can trade only if they are connected by a link. A household of type \( k \) (degree \( k \)) has links to \( k \) different firms, and each firm faces the same degree distribution among its potential customers (i.e. the households to which it is linked). One justification of this particular kind of symmetry among firms is that the network itself might be symmetric in the sense that each firm faces the same degree distribution but may in principle attract a different number of potential consumers. Alternatively it could also be the case that firms do not know the degree of each consumer they are linked to but instead only know the degree distribution in the overall network.\(^7\) The incomplete information setup seems to be realistic approximation of situations where firms normally do not know the shopping habits of each individual consumer, but rely their strategy on survey data representing the shopping habits of average customers instead.

We will use both – the search and the network – interpretation interchangeably; in either case, we obtain a strategic market game among the \( N \) firms, where we can take without loss of generality the strategy set of each firm to be the unit interval \([0, 1]\) (a price below 0 would generate losses, and at a price above 1 nobody would buy). Trivial cases apart, we will see that this game has equilibria only in mixed strategies, thus generating price dispersion. Our main interest is the dependence of these price dispersion equilibria, and hence consumer welfare, on the information structure \( q \).

First we introduce some more terminology and notation. A household of type \( k = 1 \) is called uninformed (or locked in in the network case), households of types \( k \geq 2 \) are called informed (also searchers or shoppers). The average number of searches (or links) per household is \( \kappa = E[q] = \sum_k kq_k \); it is a measure for the intensity of search in the market (density of the network); the number of links per firm is \( \mu \kappa \). For \( q = (q_1, \ldots, q_N) \), define the auxiliary functions

\[
\varphi(x) := \sum_k kq_k x^{k-1} \quad \text{for} \quad x \in [0, 1]
\]

and if \( q_1 > 0 \)

\[
\psi(x) := \frac{1}{q_1} \varphi(x) \quad \text{for} \quad x \in [0, 1]
\]

We have \( \psi(0) = 1, \psi(1) = \kappa/q_1 \), and \( \psi \) is continuous and strictly increasing in \( x \).

The following known results are stated here for easy reference.

**Proposition 2.1.** A market game with information structure \( q = (q_1, \ldots, q_N) \) has the following equilibria:

(a) if all households are uninformed \((q_1 = 1)\), the only equilibrium is the monopoly outcome, i.e. all firms charge the households’ reservation price \((= 1)\).

\(^7\)See footnote 6.
(b) if all households search \((q_1 = 0)\), the only equilibrium (in which all firms are active) is the competitive outcome, i.e. all firms charge the competitive price (= marginal cost = 0).

(c) If \(0 < q_1 < 1\), there is no equilibrium in pure strategies, but there exists a unique symmetric equilibrium in mixed strategies: each firm chooses its price at random according to a continuous distribution \(F(p)\) with support \([p_{\min}, p_{\max}]\), where

\[
0 < p_{\min} = \frac{1}{\psi(1)} = \frac{q_1}{\kappa} < 1, \quad p_{\max} = \frac{1}{\psi(0)} = 1
\]

and

\[
F(p) = 1 - \psi^{-1} \left( \frac{1}{p} \right) \quad \text{for } p_{\min} \leq p \leq p_{\max}
\]

(and of course \(F(p) = 0\) for \(p \leq p_{\min}\), \(F(p) = 1\) for \(p \geq p_{\max}\)). Moreover, the equilibrium profit per firm is \(\pi = \mu q_1\), and the average selling price (average household expenditure) is \(p_{\text{av}} = q_1\).

These results are known at least since Burdett and Judd (1983) (see especially the proof of Lemma 2 there); for the reader’s convenience, we give a brief sketch of the proof of Proposition 2.1(c) in the Appendix. We denote by \(K = \max\{k \in \{1, \ldots, N\} | q_k > 0\}\) the highest type actually occurring in the population, and observe that the function \(\psi\) is a polynomial of degree \(K - 1\). Therefore the equilibrium price distribution \(F\) depends only on \(K\), not on the total number of firms \(N\). Since every customer of a firm observes at most \(K - 1\) other prices, each firm has, as it were, only \(K - 1\) effective competitors, and the model behaves like an oligopoly with \(K\) firms. Thus we may take the total number \(N\) of firms as being arbitrarily large, the effective size \(K\) of the market being determined by the information structure \(q\).

From now on, we focus on price dispersion equilibria and assume always \(0 < q_1 < 1\). The equilibrium price distribution \(F\) can take different forms, depending on the information structure \(q\). To illustrate this, Fig. 2 exhibits the density \(f(p) = F'(p)\) for six different information structures in a market with \(N = 5\) firms.

The expected price \(p_k\) paid by a household of type \(k\) is given by

\[
p_k = \int_{p_{\min}}^{1} p \, dF_k(p) \quad (k = 1, \ldots, N)
\]

where \(F_k(p) = 1 - [1 - F(p)]^k\) is the distribution of the minimum of a sample of size \(k\) from the distribution \(F\). It is easy to see\(^8\) that the expected price \(p_k\) can also be written \(p_k = \int_{p_{\min}}^{1} [1 - F(p)]^k \, dp\). This implies the well-known fact that \(p_k\) is a strictly decreasing, convex function of \(k\) (cf. Burdett and Judd (1983), p. 961). We write \(p = (p_1, \ldots, p_N)\) for the list of expected prices paid by the various types.

The next lemma will turn out to be very useful for the comparative statics exercise presented in the next section.

**Lemma 2.2.** The expected price of a consumer of type \(k\) is given by

\[
p_k = \int_{0}^{1} \frac{1}{\psi(x)} \, dx = \int_{0}^{1} \frac{k \cdot x^{k-1}}{\psi(x)} \, dx. \tag{1}
\]

\(^8\)\(p_k = \int p \cdot F_k'(p) \, dp = \int pk [1 - F(p)]^{k-1} F'(p) \, dp = \int pk [1 - F(p)]^{k-1} \, dF(p) = \int [1 - F(p)]^k \, dp\), where the last equality follows by partial integration.
Proof: see Appendix.

Using this lemma, we can check that the average of the $p_k$’s is indeed equal to the average selling price $p_{av} = q_1$:

$$q \cdot p = \sum_k q_k p_k = \int_0^1 \frac{\sum_k q_k k u^{k-1}}{\psi(u)} du = \int_0^1 \frac{\varphi(u)}{\psi(u)} du = q_1 = p_{av}.$$  

We conclude this section with an analytically tractable example.

**Example 2.3.** Suppose the information structure is such that any $q_k$ is proportional to $\alpha_k$, with $\alpha \in (0, 1)$.

In order to get the normalization $\sum_k q_k = 1$, at the limit of infinitely many firms, we need $q_k = \frac{1}{-\log(1-\alpha)} \frac{\alpha_k}{k}$. By the previous Proposition 2.1 the price distribution is

$$F(p) = \frac{p}{\alpha} - \frac{1-\alpha}{\alpha},$$

for $p \in [(1-\alpha), 1]$, so that $f(p) = \frac{1}{\alpha}$ on this support (uniform probabilities), and is 0 otherwise. The expected price is $p_1 = E[f(p)] = (1 - \frac{\alpha}{2})$. We can also compute directly from (1) the expected prices paid by each customer: $p_k = (1 - \frac{k}{k+1} \alpha)$. If $\alpha$ increases, also the probabilities that customers have more connections increase, and expected prices decrease.

### 3. The Digital Divide

The boundary between households who have access to the internet and those who do not is sometimes referred to as the "digital divide", and there is evidence that households above the divide (with many links) pay lower prices on average than those below (see e.g. Baye, Morgan, and Scholten (2003)). This is trivially true in our model, too, where ever we put the "divide", simply because the expected prices $p_k$ decrease with $k$.

In what follows, we will demonstrate the existence of a much less obvious, but perhaps even more deplorable kind of divide (which, for lack of a better term, we also call "digital"): if some of those consumers who already search (have two or more links) begin to search even more (in a certain well-defined sense, see Definition 1 below), then the equilibrium price distribution changes in such a way that all types below a certain threshold (our "digital divide") face higher expected prices than before (in particular, the uninformed households always suffer), while those above the divide face lower prices than before (Theorem 3.2).

That is, (a certain way of) increasing the information in the market (making the network denser) favors only the high types, and harms the low types. Nobody here searches less than before, but the increased activity of some produces a negative (pecuniary) externality for the others (the low types who do not change their behavior). This is made precise in the following.

Consider two different search distributions $q = (q_1, \ldots, q_N)$ and $\bar{q} = (\bar{q}_1, \ldots, \bar{q}_N)$, and denote the associated equilibrium quantities by $F, \bar{F}, p_k, \bar{p}_k$, etc.

**Definition 1.** $\bar{q}$ has fewer low types and more high types than $q$ if there exists a threshold type $\ell$, $1 \leq \ell < N$, such that

$$\bar{q}_k \leq q_k \text{ for } k < \ell, \quad \bar{q}_\ell < q_\ell, \quad \bar{q}_k \geq q_k \text{ for } k > \ell.$$
In this case, we write \( \tilde{q} \succ_{\ell} q \) or simply \( \tilde{q} \succ q \), and say that households search more under \( \tilde{q} \) than under \( q \).

**Remark.** The concept just introduced is stronger than first-order stochastic dominance, i.e. \( \tilde{q} \succ q \) implies \( \tilde{q} \text{ FOSD} q \), but not vice versa. Here \( \tilde{q} \text{ FOSD} q \) means that \( \tilde{q}_1 + \cdots + \tilde{q}_k \leq q_1 + \cdots + q_k \) for all \( k = 1, \ldots, N \). It is easy to see that if \( \tilde{q} \text{ FOSD} q \) and \( \tilde{q} \neq q \), then \( k > k' \) and hence \( p_{\min} > \tilde{p}_{\min} \). Therefore this last inequality is also implied by \( \tilde{q} \succ q \).

We want to see what happens to the expected prices \( p_k \) if households search more in the sense of Definition 1, and consider the following two cases in turn. First, we consider the case when uninformed consumers start to compare more prices, and second, we consider the case when already informed consumers compare more prices.

**Uninformed Consumers begin to search.** Consider first the case that only the uninformed households (type 1) search more. This means that \( \tilde{q}_1 < q_1 \) and \( \tilde{q}_k \geq q_k \) for all \( k \geq 2 \), i.e. \( \tilde{q} \succ_{1} q \).

Here we find inline with the previous literature:

**Theorem 3.1.** Assume \( \tilde{q}_1 < q_1 \) and \( \tilde{q}_k \geq q_k \) for all \( k \geq 2 \), i.e. \( \tilde{q} \succ_{1} q \). Then \( \tilde{p}_k < p_k \) for all \( k \).

*Proof.* Obviously the assumption implies that \( \tilde{q}_k / \tilde{q}_1 \geq q_k / q_1 \) for all \( k \), with strict inequality for some \( k \), so that \( \psi(x) > \psi(x) \) for all \( x \in (0, 1) \). This in turn implies \( \tilde{p}_k < p_k \) for all \( k \) by equation (1).

That is, if some uninformed households begin to search, the expected prices for all types go down. The average selling price, \( p_{av} = q_1 \) also goes down. Intuitively, there is more search and the market becomes more competitive.

**Informed Consumers search more.** Consider next the case that only the informed households (types \( k \geq 2 \)) search more. This means that \( \tilde{q}_1 = q_1 \) and \( \tilde{q} \succ_{\ell} q \) for some \( \ell \geq 2 \). Here the situation is not so transparent: there is also “more search”, but the average selling price \( p_{av} = q_1 = \tilde{q}_1 \) remains the same. This suggests (but does not prove) that not all prices \( p_k \) can go down. Our main result is the following.

**Theorem 3.2.** Assume \( \tilde{q}_1 = q_1 \) and \( \tilde{q} \succ_{\ell} q \) for \( 2 \leq \ell < N \); and \( N \) sufficiently large. Then there exists a number \( d \), \( 1 < d < N \) such that \( \tilde{p}_k > p_k \) for \( k < d \) and \( \tilde{p}_k < p_k \) for \( k > d \).

*Proof:* see Appendix.

We call \( d \) the digital divide. All types below the digital divide pay higher expected prices, and all types above it pay lower expected prices. In particular, the uninformed households \( (k = 1) \) always suffer because \( d > 1 \). Interestingly, in the following example 3.3 all types \( k \) of consumers with \( q_k > 0 \) will expect to pay a higher price.\textsuperscript{9}

It is important to realize that firms’ profits and hence also total consumer welfare depend only on the fraction of informed consumers. More search by already informed consumers does not affect total consumer welfare. However, it amounts to a redistribution of consumer welfare from relatively uninformed consumers to relatively informed consumers. In this sense, more search by informed consumers imposes a negative externality on relatively uninformed consumers and may impose a positive externality on highly informed consumers.

\textsuperscript{9}This is possible only if \( K \leq d \).
Interestingly, we find that the number of firms does not affect consumer welfare either (as long as the search distribution $q$ does not change). Increasing the number of firms just results in lower profits per firm, leaving the industry’s profits and hence also consumer welfare unaffected. This is in contrast to previous work by Rosenthal (1980) who considers a model with uninformed and fully informed consumers and shows that if new firms enter the market all types of consumers will expect to pay a higher price. The driving force behind this surprising result is the assumption that each new firm brings new uninformed consumers to the market. As the number of firms increases also the fraction of uninformed consumers in the market increases and competition becomes less though, confirming Theorem 3.1. Morgan, Orzen, and Sefton (2006) also consider a model with uninformed and fully informed consumers and show that increasing the number of firms will decrease expected price for informed buyers and increase expected prices paid by uninformed consumers. The main reason behind this result is the assumption that the informed consumers always sample all firms in the market. As the number of firms increases also the number of prices sampled by the informed consumers increases. Morgan, Orzen, and Sefton (2006) show that in this setup uninformed consumers will expect to pay a higher price and the fully informed consumers will expect to pay a lower price, which can be readily interpreted in the light of Theorem 3.2.

We will now present an example where some consumers who previously compared two prices start to compare three prices.

**Example 3.3.** Consider the case where consumers only sample at most three firms, i.e. $q_1 + q_2 + q_3 = 1$ and consider the consumer search distribution $q = (\frac{1}{3}, \frac{1}{3}, \frac{1}{2})$. Using Lemma 2.2 we can numerically compute the expected prices paid by the different types of consumers, obtaining $p_1 \simeq 0.3761$, $p_2 \simeq 0.2408$ and $p_3 \simeq 0.1915$. Suppose now that a mass of $\frac{1}{20}$ of consumers who previously sampled two firms sample now three firms, obtaining a new consumer search distribution $\tilde{q} = (\frac{1}{4}, \frac{1}{5}, \frac{11}{20})$. Computing expected prices under this new consumer search distribution we now obtain that $\tilde{p}_1 \simeq 0.3832 > \tilde{p}_1$, $\tilde{p}_2 \simeq 0.2434 > p_2$ and $\tilde{p}_3 \simeq 0.1919 > p_3$. In addition, we find that a consumer of type 4 would actually pay a lower price (i.e. $\tilde{p}_4 < p_4$), but there are no such consumers represented in the population.

At a first glance, the previous example seems to be in contradiction with the observation that as the number of uninformed consumers is the same under both consumer search distributions also the expected selling price has to be constant. This contradiction is however easily resolved if one takes into account that now those consumers who switched from sampling two firms to three firms now pay a lower price, i.e. $\tilde{p}_3 < p_2$.

**4 Conclusion**

It has become folk wisdom in economics that consumer welfare can be increased in search models by facilitating consumer search. If previously uninformed consumers start to compare prices this is definitely the case. However, if already informed consumers compare more prices total consumer welfare stays unaffected. Moreover, it will result in a redistribution of consumer welfare from relatively uniformed to informed consumers.

The internet – and with it the onset of price comparison sites – has been largely praised by economists and policy makers as a way to ease consumer search and thereby increase competition. Consequently,

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10 Similar observations are known as Simpson’s paradox in statistics.
policies to improve internet accessibility, and to promote the use of price comparison sites, were also seen as tools to increase consumer welfare. Whether such policies are indeed beneficial to all consumers is however questionable. Suppose there are three types of consumers: those who are uninformed consumers, those who sample just a few prices, e.g. by using newspapers ads, and those who rely on internet price comparison sites. If the uninformed consumers won’t be affected by measures promoting search, total consumer welfare will remain constant. Moreover, if partially informed consumers decide to become fully informed, then the uninformed will expect to pay a higher price. Consequently, when trying to enhance competition by easing the burden of comparing prices uninformed consumers should be targeted first.

References


11 As seen in Example 3.3, it can even be the case that all types of consumers expect to pay a higher price.


5. Appendix

*Sketch of Proof of Prop. 2.1(c).*

First one establishes that the equilibrium price distribution $F$ must be continuous and strictly increasing on a support of the form $[p_{\text{min}}, p_{\text{max}}]$ with $0 < p_{\text{min}} < p_{\text{max}} = 1$ (otherwise, one can easily find profitable deviations). Given this, we observe that every price in the support must give the same (expected) payoff to a firm. If the firm charges $p_{\text{max}} = 1$ it gets only its share of uninformed consumers, so $\pi = p_{\text{max}} \cdot \mu q_1 = \mu q_1$ is the equilibrium profit. If it charges $p_{\text{min}}$ it gets all households to which it is linked, so $\pi = p_{\text{min}} \cdot \mu \kappa$, hence $p_{\text{min}} = q_1 / \kappa$. Total profits are $N \pi$ and must be equal to total household expenditure $M_{\text{av}}$, so $p_{\text{av}} = (N / M) \mu q_1 = q_1$.

A firm charging any price $p \in [p_{\text{min}}, 1]$ is observed by $\mu q_k$ households of type $k$ and makes a sale to such a household iff the other $(k - 1)$ prices observed by the household are higher than $p$, which occurs with probability $[1 - F(p)]^{k-1}$. Thus the firm’s profit is

$$ p \cdot \sum_{k=1}^{N} \mu q_k [1 - F(p)]^{k-1} = p \cdot \mu q_1 \cdot \psi[1 - F(p)] = \pi = \mu q_1 $$

This implies

$$ p \cdot \psi[1 - F(p)] = 1 $$

or $F(p) = 1 - \psi^{-1}(1/p)$.

**Proof of lemma 2.2.** The function $x = F_k(p) = 1 - [1 - F(p)]^k$ is one-to-one from $[p_{\text{min}}, 1]$ to $[0, 1]$. Solving for $p$ gives the inverse function $p = F_k^{-1}(x)$ as follows: $(1 - x)^{1/k} = 1 - F(p) = \psi^{-1}(1/p)$ (by Prop. 2.1), hence $1/p = \psi[(1 - x)^{1/k}]$ or

$$ p = F_k^{-1}(x) = \frac{1}{\psi[(1 - x)^{1/k}]} $$

It is geometrically obvious that the areas in the unit square to the left and to the right of the graph of $F_k$ (which is the same as the graph of $F_k^{-1}$) sum to one, i.e. $\int_0^1 F_k^{-1}(x) \, dx + \int_{p_{\text{min}}}^1 F_k(p) \, dp = 1$. By a well-known formula for RS-integrals we have

$$ \int_{p_{\text{min}}}^1 p \, dF_k(p) + \int_{p_{\text{min}}}^1 F_k(p) \, dp = p \cdot F_k(p)|_{p_{\text{min}}}^1 $$
The leftmost term in this expression is \( p_k \), and the RHS is equal to 1, hence
\[
p_k = 1 - \int_{p_{\min}}^{1} F_k(p) \, dp = \int_{0}^{1} F_k^{-1}(x) \, dx = \int_{0}^{1} \frac{1}{\psi((1-x)^{1/k})} \, dx
\]
The change of variable \( x = f(u) := 1 - u^k \), with \( f'(u) = -ku^{k-1} \), transforms the last integral to
\[
p_k = \int_{0}^{1} \frac{ku^{k-1}}{\psi(u)} \, du = \int_{0}^{1} \frac{1}{\psi(x)} \, dx^k
\]
\[\square\]

The proof of Theorem 3.2 is preceded by some lemmas.

**Lemma 5.1.** Assume \( \bar{q}_1 = q_1 \) and \( \bar{q} > \ell \, q \), \( 2 \leq \ell < N \). Then there is a number \( b \in (0,1) \) such that \( \psi(x) < \psi(x) \) for \( 0 < x < b \) and \( \psi(x) > \psi(x) \) for \( b < x \leq 1 \).

Of course, by continuity, \( \psi(b) = \psi(b) \), i.e., \( \psi \) and \( \psi \) cross only once in \((0,1]\), at the point \( b \).

**Proof of Lemma 5.1:** Since \( \bar{q}_1 = q_1 > 0 \), it suffices to prove the assertion for \( \varphi = q_1 \psi \). By assumption, \( q_k - \bar{q}_k \geq 0 \) for \( k < \ell \), \( q_\ell - \bar{q}_\ell > 0 \), and \( q_k - \bar{q}_k \leq 0 \) for \( k > \ell \).

We know that \( \varphi(0) - \bar{\varphi}(0) = q_1 - \bar{q}_1 = 0 \) and \( \varphi(1) - \bar{\varphi}(1) = \kappa - \bar{\kappa} < 0 \). For \( 0 < x \leq 1 \) we have
\[
\varphi(x) - \bar{\varphi}(x) = \sum_{k=1}^{\ell} k(q_k - \bar{q}_k) x^{k-1} + \sum_{k=\ell+1}^{N} k(q_k - \bar{q}_k) x^{k-1} = \frac{1}{x^{\ell-1}} [A(x) - B(x)]
\]
where
\[
A(x) = \sum_{k=1}^{\ell-1} k(q_k - \bar{q}_k) \frac{1}{x^{k-1}} + \ell(q_\ell - \bar{q}_\ell)
\]
is nonincreasing in \( x \) and \( \geq \ell(q_k - \bar{q}_k) > 0 \forall x \), and
\[
B(x) = \sum_{k=\ell+1}^{N} k(q_k - \bar{q}_k) x^{k-\ell}
\]
is strictly increasing in \( x \) (because at least one coefficient \( k(q_k - \bar{q}_k) \) must be positive), and tends to zero for \( x \to 0 \). Therefore the function \( f(x) := A(x) - B(x) \) is strictly decreasing on \((0,1]\), positive for \( x \) near zero, and negative for \( x = 1 \) (because \( \varphi(1) - \bar{\varphi}(1) < 0 \)). This implies the assertion. \[\square\]

**Lemma 5.2.** Let \( g(x) \) be a nonnegative continuous function on the interval \([0,1]\) which is strictly positive except in at most finitely many points, and choose \( b \) with \( 0 < b < 1 \). For \( i = 1, 2, \ldots \) define
\[
A_i := \int_{0}^{b} g(x) \, dx^i = \int_{0}^{b} g(x) x^{i} \, dx, \quad B_i := \int_{0}^{1} g(x) \, dx^i = \int_{b}^{1} g(x) x^{i} \, dx.
\]
Then
\[
\frac{B_i}{A_i} < \frac{B_{i+1}}{A_{i+1}} \quad \text{for} \quad i = 1, 2, \ldots
\]
(Obviously \( A_i, B_i \) are always positive).
Intuitively, the distribution \( H_i(x) = x^i \) one \([0,1]\) has more weight on the right if \( i \) increases, hence \( B_i \) should increase relative to \( A_i \).

**Proof of Lemma 5.2:** Fix \( b \in (0,1) \) and \( i \geq 1 \). For any \( n \geq 2 \) the integrals \( A_i \) and \( B_i \) can be written as Riemann sums

\[
A_i = \sum_{j=1}^{n} g(\eta_j) i^n \eta_j^{i-1} d\eta_j, \quad B_i = \sum_{j=1}^{n} g(\xi_j) i^n \xi_j^{i-1} dz_j
\]

where

\[
0 = y_0 < y_1 < \cdots < y_n = b, \quad b = z_0 < z_1 < \cdots < z_n = 1
\]

\( \eta_j \in (y_{j-1},y_j), \quad \xi_j \in (z_{j-1},z_j), \quad dy_j = y_j - y_{j-1}, \quad dz_j = z_j - z_{j-1} \quad \forall j \)

Moreover, the numbers \( y_j, \eta_j, z_j, \xi_j \) can be chosen so that the summands

\[
\alpha_{i,j} := g(\eta_j) i^n \eta_j^{i-1} d\eta_j = \int_{y_{j-1}}^{y_j} g(x) i^n x^{i-1} dx
\]

are all equal, i.e. \( \alpha_{i,j} = A_i/n \) for all \( j = 1, \ldots, n \); and the summands

\[
\beta_{i,j} := g(\xi_j) i^n \xi_j^{i-1} dz_j = \int_{z_{j-1}}^{z_j} g(x) i^n x^{i-1} dx
\]

are also all equal, i.e. \( \beta_{i,j} = B_i/n \) for all \( j = 1, \ldots, n \).

Now, using these same numbers \( y_j, \eta_j, z_j, \xi_j \), we can approximate \( A_{i+1} \), \( B_{i+1} \) by Riemann sums of the form

\[
\tilde{A}_{i+1} = \sum_{j=1}^{n} \alpha_{i+1,j}, \quad \tilde{B}_{i+1} = \sum_{j=1}^{n} \beta_{i+1,j}
\]

where

\[
\alpha_{i+1,j} := g(\eta_j) (i+1) \eta_j^{i+1} d\eta_j, \quad \beta_{i+1,j} := g(\xi_j) (i+1) \xi_j^{i+1} dz_j.
\]

By choosing \( n \) is sufficiently large, we can make \( \tilde{A}_{i+1} \) resp. \( \tilde{B}_{i+1} \) arbitrarily close to \( A_{i+1} \) resp. \( B_{i+1} \) (because \( g \) is continuous and all possible Riemann sums converge to the same integral if the grid size goes to zero). Then

\[
\beta_{i+1,j} \over \alpha_{i+1,j} = \frac{g(\xi_j) (i+1) \xi_j^{i+1} dz_j}{g(\eta_j) (i+1) \eta_j^{i+1} d\eta_j} = \frac{\beta_{i,j} \xi_j}{\alpha_{i,j} \eta_j} = \frac{B_i \xi_j}{A_i \eta_j}
\]

(3)

By construction, \( \xi_j/\eta_j > z_{j-1}/y_j \geq \min_{1 \leq i \leq n} \{z_{i-1}/y_i\} =: \lambda > 1 \) (recall \( n \geq 2 \)). Moreover, if we proceed to a finer partitioning, replacing \( n \) by \( n' = n + 1 \) (with the same properties as above, i.e. all summands \( \alpha_{i,j}' \) resp. \( \beta_{i,j}' \) are equal), then \( y_j' < y_j \) and \( z_j' > z_{j-1} \) for \( j = 1, \ldots, n \). Therefore a fortiori \( \xi_j'/\eta_j' > z_{j-1}'/y_j' > z_{j-1}/y_j \geq \lambda > 1 \).

Equation (3) implies \( \beta_{i+1,j} > \lambda (B_i/A_i) \alpha_{i+1,j} \), hence

\[
\tilde{B}_{i+1} = \sum_{j=1}^{n} \beta_{i+1,j} > \lambda \frac{B_i}{A_i} \sum_{j=1}^{n} \alpha_{i+1,j} = \lambda \frac{B_i}{A_i} \tilde{A}_{i+1}
\]

Therefore \( \tilde{B}_{i+1}/\tilde{A}_{i+1} > \lambda B_i/A_i \) and, since \( \tilde{B}_{i+1}/\tilde{A}_{i+1} \to B_{i+1}/A_{i+1} \) as \( n \to \infty \), we obtain \( B_{i+1}/A_{i+1} \geq \lambda B_i/A_i > B_i/A_i \). \( \blacksquare \)
Proof of Theorem 3.2. Let $\tilde{q}_1 = q_1, \tilde{q} \succ \ell q, 2 \leq \ell < N$, and denote by $p_k$ resp. $\tilde{p}_k$ the expected price paid by a type $k$ household under the search distribution $q$ resp. $\tilde{q}$, for $k = 1, 2, \ldots N$.

Consider the function
\[
h(x) := \frac{1}{\psi(x)} - \frac{1}{\tilde{\psi}(x)}.
\]
By Lemma 5.1, there is a point $b$, $0 < b < 1$, such that $h(x) < 0$ for $0 < x < b$ and $h(x) > 0$ for $b < x < 1$. Therefore, by equation (1):
\[
p_k - \tilde{p}_k = \int_0^1 h(x) dx = B_k - A_k
\]
where $A_k = \int_0^b |h(x)| dx$, $B_k = \int_b^1 |h(x)| dx$. The function $g(x) := |h(x)|$ satisfies the assumptions of Lemma 2, hence $B_k/A_k$ increases strictly with $k$. Thus, if $p_m - \tilde{p}_m = B_m - A_m > 0$ for some $m$, so that $B_m/A_m > 1$, we have also $B_k/A_k > 1$, hence $p_k - \tilde{p}_k = B_k - A_k > 0$ for all $k \geq m$. In other words, if the expected price decreases for some type $m$, then also for all higher types.

Denote by $d$ the lowest type for whom the expected price does not increase, i.e. $d$ is the first index such that $p_d \geq \tilde{p}_d$.

We know that $\tilde{p}_{\min} < p_{\min}$, therefore $\tilde{p}_N < p_N$ for $N$ sufficiently large (because a household who searches long enough must find a price arbitrarily close to the minimum); thus $d < N$ for $N$ sufficiently large.

It remains to show that $d > 1$, i.e. not all prices go down. Using vector notation, write $q \cdot p = \sum_k q_k p_k$.

Then the expected average prices satisfy
\[
\tilde{q}_1 = \tilde{p}_{\text{av}} = \tilde{p} \cdot \tilde{q} = q_1 = p_{\text{av}} = p \cdot q
\]
Moreover, the components of $p = (p_1, \ldots, p_N)$ are strictly decreasing and $\tilde{q}$ FOSD $q$. Therefore $\tilde{q} \cdot p < q \cdot p$. If all prices $\tilde{p}_k$ were less than $p_k$, we should have $\tilde{p} \leq p$, hence $\tilde{q} \cdot \tilde{p} \leq \tilde{q} \cdot p < q \cdot p$, contradicting (4).
Figure 2: Some densities $f(p)$ for various $q = (q_1, \ldots, q_s)$, with support $[p_{\min}, 1]$. 