Every day thousands of decisions are made by all kinds of committees, parliaments, councils and boards by a ‘yes–no’ voting process. Sometimes a committee can only accept or reject the proposals submitted to it for a decision. On other occasions, committee members have the possibility of modifying the proposal and bargaining an agreement prior to the vote. In either case, what rule should be used if each member acts on behalf of a different-sized group? It seems intuitively clear that if the groups are of different sizes then a symmetric rule (e.g. the simple majority or unanimity) is not suitable. The question then arises of what voting rule should be used. *Voting and Collective Decision-Making* addresses this and other issues through a study of the theory of bargaining and voting power, showing how it applies to real decision-making contexts.

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To my parents
Annick

For loved ones gone and those still around
Federico
# Contents

<table>
<thead>
<tr>
<th>List of figures</th>
<th>page x</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preface</td>
<td>xi</td>
</tr>
</tbody>
</table>

## 1 Preliminaries

1.1 Basic set-theoretic notation 1

1.2 Some combinatorics 2

1.2.1 Permutations and combinations 2

1.2.2 Some useful approximations 3

1.3 Voting rules 4

1.3.1 Dichotomous voting rules 5

1.3.2 Some particular voting rules 7

1.4 Expected utility theory 10

1.4.1 Players, games and game theory 10

1.4.2 Preferences and utility 10

1.4.3 Lotteries and expected utility 11

1.4.4 Expected utility preferences 13

1.5 Some basic game theory notions 18

1.5.1 Equilibrium 19

1.5.2 Cooperative and non-cooperative game theory 20

1.5.3 Subgame perfect equilibrium 21

1.5.4 Basic cooperative models 24

1.6 Exercises 26

## 2 Seminal papers, seminal ambiguities

2.1 Seminal papers and seminal ambiguities 30

2.1.1 Nash (1950): The bargaining problem 30

2.1.2 Shapley (1953): The value of a TU game 34

2.1.3 Shapley–Shubik (1954): A power index 37

2.1.4 Banzhaf (1965): Power as decisiveness 39

2.1.5 Penrose (1946), Rae (1969) and Coleman (1971) 41
2.1.6 Through the axiomatic glasses: Dubey (1975), Dubey–Shapley (1979) 41
2.2 Clear-cut models to dissipate ambiguity 44
2.3 Further reading 46
2.3.1 Axiomatic approach 47
2.3.2 Probabilistic approach 48
2.4 Exercises 48

3 ‘Take-it-or-leave-it’ committees 52
3.1 The take-it-or-leave-it scenario 52
3.2 Success and decisiveness in a vote 54
3.3 Preferences, behaviour and probabilities 55
3.4 Success and decisiveness ex ante 57
3.5 A priori assessments based on the voting rule 60
3.5.1 Rae index 62
3.5.2 Banzhaf(–Penrose) index 62
3.5.3 Coleman indices 63
3.5.4 König and Bräuninger's inclusiveness index 65
3.5.5 Summary and remarks 65
3.6 Success versus decisiveness 67
3.6.1 Success is the issue in a take-it-or-leave-it scenario 67
3.6.2 Conditional success 69
3.6.3 Summary 70
3.7 The choice of voting rule: egalitarianism and utilitarianism 71
3.7.1 Egalitarianism 74
3.7.2 Utilitarianism 74
3.8 The choice of voting rule in a committee of representatives 77
3.8.1 An ideal two-stage decision procedure 78
3.8.2 Egalitarianism in a committee of representatives 81
3.8.3 Utilitarianism in a committee of representatives 87
3.9 Exercises 97
3.10 Appendix 99
Contents

4 Bargaining committees 105
  4.1 The bargaining scenario 106
  4.2 A model of a bargaining committee: voting rule and voters’ preferences 107
  4.3 Cooperative game-theoretic approach 109
    4.3.1 Rationality conditions 110
    4.3.2 Axiomatic characterizations 112
    4.3.3 Discussion 116
  4.4 A non-cooperative model of a bargaining committee 117
    4.4.1 Probabilistic protocols 119
    4.4.2 Bargaining protocols under a voting rule 123
    4.4.3 Discussion 127
  4.5 Egalitarianism and utilitarianism in a bargaining committee 128
  4.6 The neutral voting rule in a committee of representatives 130
  4.7 Exercises 134

5 Application to the European Union 136
  5.1 Voting rules in the European Council 136
  5.2 The Council as a take-it-or-leave-it committee 142
    5.2.1 Criteria based on probabilities 143
    5.2.2 Criteria based on utilities 156
  5.3 The Council as a bargaining committee 163
  5.4 Exercises 170

Conclusions 172
References 176
Index 182