Local Public Good Provision in a Segregated Society*

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Abstract
In this paper we analyze the decision of two segregated jurisdictions who can either provide a public good jointly or separately. We focus on the trade-off between economies of scale and preference heterogeneity where the preference heterogeneity has two components: the preference over the location of the public good and the preference over the group composition. We obtain that if the communities care equally about the composition of the jurisdiction, the consolidation takes place between relatively similar sized communities. When the smaller community cares more about the composition, consolidation will occur if it has a relatively moderate size. We also obtain that democracy leads to an excessive separation, from a social point of view and in case of a non-excludable public good, when people care more about group composition, free-riding is less likely.

1 Introduction
Neighboring jurisdictions may decide to provide a public facility together or separately.¹ There exists a huge literature on the unification or secession decision of jurisdictions and the stability of the grand coalition. As in this study, in general the main trade-off is between economies of scale and heterogeneity. Larger jurisdictions enjoy larger economies of scale but at the same time the heterogeneity costs are higher in them. However, in this paper, we also analyze the preferences over composition of beneficiaries which means that individuals care about with whom they share the public good. How the approval and efficiency conditions for unification is affected from this kind of preferences is the first question that is addressed in this study. Obviously, joint provision of the

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¹Various terms are used to refer joint provision such as unification, annexation or consolidation.
public good is more costly and so less likely. However, although the approval conditions get more strict, their characteristics will be affected since the changes will not be equal for each jurisdiction. Moreover, free-riding and the location decision policies will change interestingly.

Consolidation, i.e., cooperation in the provision of a local public good or service, while providing other public facilities separately, is an important mechanism in determining and shaping the number and size of jurisdictions. It is a process by which distinct jurisdictions share the costs and benefits of a new public good. There are many jurisdictions that have separated fire departments and park services while cooperating in public schooling services with a neighboring jurisdiction. For instance, in less than one century the number of school districts in the United States has fallen from 125,000 to 15,500 through consolidation. (Wiles, 1994)

The decision of consolidation/unification for the public good depends on the trade-off between economies of scale and heterogeneity. Larger populations have larger scale benefits but also the costs of increased heterogeneity. In this paper, our interest is focused on the decision of consolidation or independent provision of a public good in two different sized jurisdictions in which individuals also care about the composition of the population of beneficiaries. Consider, for example, the potential problem of consolidating school districts of two jurisdictions. Larger districts have economies of scale and therefore the advantage of being able to provide libraries, sports facilities, and administration on a district-wide basis with a low cost per resident. On the other hand, when consolidation occurs, i.e. a larger district created, an agreement must be reached between an increased number of individuals on common educational policies or on the location of the school. Moreover, families have to agree to mix their children with the children of another group. If prejudices exist with regard to the other group due to some ethnic, racial, or historical reasons, an individual’s utility declines upon mixing. That is to say, if an increase in size implies an increase in heterogeneity, there may be an important trade-off in terms of utility for group members.

This paper can also be related to country formation literature. The decision of unification/separation of two regions can be analyzed by the model used in this paper. Unification in the provision of the public good, which is government, implies the benefits of large jurisdiction but also the resulting costs of heterogeneity. Residents of a region may prefer not to unify with the other region not only because of an increase in average political differences in the society but also because of preferences regarding the composition of population. We believe that by taking into consideration preferences regarding population composition in country formation, more reliable and deeper results are obtained.

There are some empirical results in the literature that support considering

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2 The unification/separation decisions between Montenegro-Serbia on May 21st 2006 and South Cyprus - North Cyprus on April 24th, 2004 were taken by a referendum. Recently, Kosova became independent by one-sided approval by referendum.

3 For example, we believe that only an economical explanation is not enough for the opinion of some European countries about Turkey’s membership to EU.
preferences over heterogeneity in race, ethnicity, or religion. Alesina et al. (2004) find a significant positive effect that racial heterogeneity consistently has on the number of local jurisdictions. Brasington (2003) concludes as "the reasons why whiter communities dislike educating their kids with those of less white communities is beyond the realm of economics. The reasons why darker communities dislike educating their kids with those of whiter communities of similar income are also subject to speculation. But these sets of preferences discourage consolidation." Although Austin (1999) focuses on the provision of public municipal annexation for provision of all local public services rather than consolidation for only one public facility, his conclusion is that cities prefer annexing surrounding communities according to properties of population composition.

Like many others, we adapt the Hotelling (1929) location framework in order to represent the heterogeneity of preferences among voters over the provision of public goods.\textsuperscript{4} With some others, this framework is mostly similar to Alesina and Spolaore (1998) where they discuss the optimality and stability of the equilibrium number of countries in different politico-economic regimes. Differently from Alesina and Spolaore but similarly to Goyal and Staal (2004), here we concentrate on the effects of jurisdictions' sizes to consolidation decision in an exogenously segregated society. Although Goyal and Staal's paper has a very similar basis, most of our results are different since we also focus on the preferences over beneficiaries' composition and its effects on specific situations. One of the pioneering works on the effect of size in consolidation decision is Ellingsen (1998). He predicts that under some conditions large entities want to consolidate with small ones, but small entities might not want to consolidate with big ones if they have the free riding option. Brasington (2003) empirically examines the role of income and racial differences on the school district consolidation decision and finds significant relations.

In our attempt to explain the consolidation decision, we consider a segregated society with two jurisdictions where an excludable public good will be provided. We call the society as segregated since in one jurisdiction we have only one community's members.\textsuperscript{5} Jurisdictions have the opportunity to provide the public good independently or jointly. That will be decided by voting. The public good has a fixed cost and it is financed by a lump sum tax from its beneficiaries. When two jurisdictions consolidate, individuals are happy to be sharing the cost of the good since they pay lower taxes. However, in the case of consolidation, the society becomes less homogenous. There are two reasons why individuals might prefer homogeneity. First, individuals who are from a similar ethnic background, race, religion or income level may have more similar preferences over a public good than the individuals from a different one. The other reason is that while sharing the public good, an individual may prefer to interact with people from her community. That is, individuals also care about the composition of the group with whom they share the public good. This be-


\textsuperscript{5}In our analysis for analytical simplicity we deal with totally segregated societies. However, the results can be generalized to mixed jurisdictions as well.
comes especially important in looking at highly diverse populations’ decisions. For example, people in their school choice do not only take into consideration its location or educational system, but also the composition of the student body. This fact in turn affects the decisions and well being of the society as a whole.

Current study first looks at each jurisdiction specific decision over consolidation. However, different from the existing literature, this paper analyzes the effects and results of preferences over the composition of beneficiaries while deciding for consolidation in provision of a public good. If the residents of a jurisdiction loose more utility because of sharing the public good with the other jurisdiction, the consolidation is less likely. Different from the literature we conclude that depending on the values of public good cost and importance of group composition, the consolidation is more likely to occur when the small jurisdiction has a moderate or similar size compared to large jurisdiction. Similar to common results, we conclude that the consolidation decision increases the total utility of the society if and only if the small jurisdiction is small enough.

We also point out how the location decision of a social planner, solvability of a disagreement and free-riding decision of small sized jurisdiction are affected by the importance of beneficiaries’ composition. When a jurisdiction cares more about the composition of the beneficiaries, the social planner’s decision for the location of the public good changes in a way that makes the jurisdiction worse off. We also consider the free-riding option and show that the minority is keener to provide their own public good when they care more about the composition and the larger jurisdiction may take a strategic decision to make the minority provide their own public good.

This paper is organized as follows. In the next section we present the model. In Section 2.3, we look for the necessary conditions under which jurisdictions will prefer to consolidate or to provide the public good independently. In Section 2.4, we characterize the conditions for approval of consolidation. We provide also the conditions to guarantee that consolidation is an improvement, i.e., the total utility of society increases when jurisdictions consolidate. Section 2.5 introduces some extensions with a glance to the social planner’s decision and Section 2.6 introduces the implications of free-riding option. Section 2.7 concludes.

2 The Model

We consider a continuum of individuals that have preferences over the location of a public good. The set of possible locations is a one-dimensional space, without loss of generality, given by the interval $I = [0, 1]$. Citizens’ preferences are single-peaked and each citizen $i$ is identified with his peak point $x_i \in I$. Individuals are uniformly distributed and immobile.

We assume that there are two communities $W$ and $B$ that are separated by an exogenous boundary at $\lambda$, i.e., they have measure of $1 - \lambda$ and $\lambda$, respectively. The community $W$ is the one that is located to the right of $\lambda$ and obviously the community $B$ is located to the left of $\lambda$. We assume that $0 < \lambda < \frac{1}{2}$, which means that the population of the type $W$ is larger than $B$. 
Each individual has an income $y$ and gets utility from both private and public consumption. After paying the tax of the public good $t_i$, income is used for private consumption $y - t_i$. We assume that the cost of public good is $K$ for each jurisdiction. Then, to provide and to benefit from a public good, individual $i$ must pay a cost that depends on the size of the jurisdiction. The tax gathered in each jurisdiction should cover the total cost of the public good. The budget constraint for a jurisdiction $j$ ($j \in \{B, W\}$) with size $S_j$ is

$$\int_{S_j} t_i \, dx = K$$

With an equal sharing scheme, where all individuals of the same jurisdiction pay an equal tax for the provision of the public good, $t_i = \frac{K}{S_j}$. Therefore, in the case of independent provision, the tax for an individual from jurisdiction $B$ is $\frac{K}{\lambda}$, and for an individual from $W$ it is $\frac{K}{1-\lambda}$. If they consolidate, all individuals in the society will pay the same tax, which is $K$.

The utility derived from the public consumption is determined by the function

$$Z_i (\beta_i, r_i) (1 - d_i)$$

where $d_i$ is the distance between the ideal point of individual $i$ and the public good and $Z$ is a function determining the utility depending on the composition of the group. The variable $\beta_i$ is the proportion of individual $i$'s community in $i$'s jurisdiction and $r_i$ is the degree of importance of sharing the public good with the other community for individual $i$. $r_i$ can also be called as intolerance factor of individual $i$. The intolerance factor $r_i$ is positive and takes the same value for all members of individual $i$'s community.

For each individual, the utility obtained from the public good is decreasing with the distance from the public good to her location. Moreover, we assume that individuals prefer to share the good with the ones that are from their jurisdiction, that is, the utility of an individual also depends on the combination of the group with whom she shares the public good. For instance, a type $B$ individual’s utility decreases as the proportion of the $B$ population in the jurisdiction decreases and vice versa. This effect is covered by the function $Z$ which is increasing in $\beta_i$ and decreasing in $r_i$. Note that the function $Z$ determines the maximum utility that an individual gets if the public good is located at his peak point ($d_i = 0$).

The utility an individual obtains from the public good is directly affected by a change in the composition of beneficiaries. An individual, when choosing among two public goods that are equally distant to her, chooses the one with the composition in which there are more beneficiaries from her community. Moreover, among two public goods with the same composition, she prefers the

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6 The results are robust even if we assume that the cost of public good is $K + kS_j$ for jurisdiction $j$ with the size $S_j$ where to provide a public good, individual $i$ must pay a cost, which includes a fixed part $K > 0$ and a variable part that is proportional to the size of the group $S_j$ with $k > 0$.  

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closer one. Furthermore, in this framework as the group composition gets more heterogeneous, the effect of changes in distance on individuals’ utility is smaller. If the population composition is more heterogeneous, an individual needs the public good closer to get the same utility as he would get in a homogenous population.

Then we can define the total utility of the individuals as

\[ U_i = y - t_i + Z_i (\beta_i, r_i) (1 - d_i) \] (2)

Finally we assume that if the jurisdiction contains only one type of individuals, or if the individuals do not care about the composition of the jurisdiction that they belong to, the utility obtained from the public good only depends on the location of the public good, i.e., if \( \beta_i = 1 \) and/or \( r_i = 0 \), the \( Z \) function is the same and fixed. The most simple form that satisfies the above mentioned condition is \( \beta_i Z \) with \( Z(1, r_i) = Z(\beta_i, 0) = 1 \).

So, given the fixed boundary at \( \lambda \), in a united society \( \beta_B = \lambda \) and \( \beta_W = 1 - \lambda \) and if two regions separate \( \beta_B = 1 \) and \( \beta_W = 1 \). Then, if two jurisdictions consolidate, the utility of an individual \( \epsilon \in [0, \lambda] \) is equal to \( y - K + \lambda r \) \( (1 - d_i) \) and the utility of an individual \( \epsilon \in [\lambda, 1] \) is \( y - K + (1 - \lambda) r \) \( (1 - d_i) \). If they decide to provide the public good independently, the utility of an individual \( \epsilon \) is \( y - \frac{K}{N} + (1 - d_i) \) for \( \epsilon \in [0, \lambda] \) and \( y - \frac{K}{N} + (1 - d_i) \) for \( \epsilon \in [\lambda, 1] \).

Each jurisdiction will make the decision to provide the public good independently or jointly. The consolidation decision is taken according to votes of individuals. We check two approval conditions; majority approval and unanimity approval. For majority approval, more than half of the population should be in favor of consolidation and for unanimity approval, vote of each individual is needed. If the majority (or unanimity) in both jurisdictions are in favor of providing the public good jointly, they consolidate and there will be one public good provided. If one of the jurisdictions does not approve the consolidation, each jurisdiction provides its own public good. Then, the type (location) of the public good is decided by majority voting. We assume that individuals are forward looking and rational, that is, they are able to foresee the outcome of the consolidation or separation. Thus, the result is a subgame perfect equilibrium of a two stage game where in the first stage individuals vote for consolidation and in the second stage they decide on the location of the public good.

3 Main Results

In this framework, where we have individuals with single peaked preferences in a single dimensional space, the median voter theorem applies, i.e., the public good will be located at median voter’s most preferred point. Since we have a uniform distribution which means that the median voter is in the center of the each neighborhood, the location of the public good will be the mid-point of the
jurisdiction (for jurisdiction $B$ it is $\frac{1}{2}$ and for $W$ it is $\frac{1+\lambda}{2}$) and if two jurisdictions consolidate, the public good will be located at $\frac{1}{2}$.

### 3.1 Median Voter’s Decision

An individual votes for consolidation if and only if its payoff is higher under consolidation. We introduce the approval condition as a threshold. An approval threshold for an individual is the minimal amount of public good cost deemed necessary to make that individual better off under consolidation. In this section, we analyze the approval conditions of the individuals in each community.

We first look for the approval threshold of the median voter of the small jurisdiction. We know that the median voter of the jurisdiction $B$, that is the individual at $\frac{1}{2}$, prefers consolidation if and only if his payoff is higher, that is

$$y - K + \lambda \rho \left( 1 - \left( \frac{1}{2} - \frac{\lambda}{2} \right) \right) > y - \frac{K}{\lambda} + 1$$

so we have the approval threshold for individual at $\frac{1}{2}$

$$K > \frac{\lambda}{1-\lambda} \left[ 1 - \lambda \rho \left( \frac{1}{2} + \frac{\lambda}{2} \right) \right] \equiv K_{\frac{1}{2}}.$$

Now we check the approval conditions for the individuals that are located in the interval $[0, \frac{1}{2})$. That is, the condition for individual at $\frac{1}{2} - x$ where $x \in (0, \frac{1}{2}]$ is

$$y - K + \lambda \rho \left( 1 - \left( \frac{1}{2} - \frac{\lambda}{2} + x \right) \right) > y - \frac{K}{\lambda} + (1 - x)$$

then the individual located at $\frac{1}{2} - x$ prefers consolidation if and only if

$$K > K_{\frac{1}{2}-x} = \frac{\lambda}{1-\lambda} x (1 - \lambda \rho) \equiv K_{\frac{1}{2}-x}.$$

Individual at $\frac{1}{2} + \bar{x}$, where $\bar{x} \in (0, \frac{1}{2}]$, approves the consolidation if and only if

$$y - K + \lambda \rho \left( 1 - \left( \frac{1}{2} - \frac{\lambda}{2} - \bar{x} \right) \right) > y - \frac{K}{\lambda} + (1 - \bar{x})$$

By rearranging terms, we have

$$K > K_{\frac{1}{2}+\bar{x}} = \frac{\lambda}{1-\lambda} \bar{x} (1 + \lambda \rho) \equiv K_{\frac{1}{2}+\bar{x}}.$$

First of all, note that $K_{\frac{1}{2}} > K_{\frac{1}{2}-x}$ and $K_{\frac{1}{2}} > K_{\frac{1}{2}+\bar{x}}$, i.e., the approval threshold of the median voter is higher than the threshold of all his community.
members. This means, the public good cost should be higher to convince the median voter to get the approval of her for consolidation (since higher public good cost means larger economies of scale). In other words, the most separatist individual in a jurisdiction is the median voter. That is to say, when the median voter in a jurisdiction approves consolidation, all individuals in that jurisdiction also do. The above arguments lead to our first proposition:

**Proposition 1** In a society where the location of the good is determined by majority voting, the median voter of the society is the most separatist.

This result is better understood when the gains from independent provision are analyzed separately. If consolidation does not occur, not only the location of the public good provides more utility to the members but also the composition of its beneficiaries becomes more homogenous which implies higher utility to all members of the jurisdiction. However, although the gain from the change of distance is the same for the individuals in the interval \([0, \frac{\lambda}{2}]\), individuals that have higher utility, i.e., individuals closer to \(\frac{\lambda}{2}\), enjoy the change of composition more than the others.

### 3.2 Majority approval

Most of the papers with a similar framework in the literature concentrate on majority voting and take median voters’ approval as a sufficient condition for majority approval. However, because of linearity, where the individuals do not care about the composition of beneficiaries \((r_i = 0)\), the majority and unanimity conditions are equal. To see that, first, we check the approval condition of individuals that are located at \(\frac{\lambda}{2} - x\) and \(\frac{\lambda}{2} + \tilde{x}\) where \(x, \tilde{x} \in [0, \frac{\lambda}{2}]\) when \(r_B\) is equal to zero. We observe that \(K_{\frac{\lambda}{2}} = K_{\frac{\lambda}{2} - x} > K_{\frac{\lambda}{2} + \tilde{x}}\), which means that if you do not get the vote of the median voter, you do not have majority approval, but if median voter votes for consolidation, all individuals do. The same argument applies when \(r_W = 0\).

In our framework it is possible to get majority approval without getting the vote of the median voter. Since the median voter is the most separatist individual of the jurisdiction, the votes for majority approval are obtained from the individuals that are closer to the borders. To get the majority approval of jurisdiction \(B\), first observe that \(K_{\frac{\lambda}{2} - x}\) is decreasing in \(x\), in other words, if the individual located at \(\frac{\lambda}{2} - x\) is in favor of consolidation, all voters in the interval \([0, \frac{\lambda}{2} - x]\) will also prefer consolidation. Moreover, as \(K_{\frac{\lambda}{2} + \tilde{x}}\) is decreasing in \(\tilde{x}\), if the individual \(\frac{\lambda}{2} + \tilde{x}\) is in favor of consolidation, all the voters in the interval \((\frac{\lambda}{2} + \tilde{x}, \lambda]\) will be in favor of it.

Hence, for the values of \(K\) that satisfy \(K > K_{\frac{\lambda}{2} - x} = K_{\frac{\lambda}{2} + \tilde{x}}\) with \(x + \tilde{x} = \frac{\lambda}{2}\), at least half of the population in jurisdiction \(B\) is in favor of consolidation. So
if, $K = K_{\frac{1}{2}-x} = K_{\frac{1}{2}+\bar{x}}$ and $x + \bar{x} = \frac{1}{2}$ exactly the half of jurisdiction $B$ would be in favor of consolidation. Thus, to find the critical value of $K$:

$$K_{\frac{1}{2}-x} = K_{\frac{1}{2}+\bar{x}}$$

which implies that

$$x(1 - \lambda^r_B) = \bar{x}(1 + \lambda^r_B)$$

and for majority approval we also need $\bar{x} + x = \frac{1}{2}$, then replacing $\bar{x}$ with $\frac{1}{2} - x$, we get

$$x(1 - \lambda^r_B) = (\frac{1}{2} - x)(1 + \lambda^r_B)$$

that is

$$x(1 - \lambda^r_B) + x(1 + \lambda^r_B) = \frac{\lambda}{2}(1 + \lambda^r_B)$$

so we have $x = \frac{\lambda}{4}(1 + \lambda^r_B)$ and $\bar{x} = \frac{\lambda}{4}(1 - \lambda^r_B)$ which leads to the following proposition:

**Proposition 2** The majority of jurisdiction $B$ is in favor of consolidation if and only if

$$K > \frac{\lambda}{1 - \lambda} \left[ 1 - \lambda^r_B \left( \frac{1}{2} + \frac{\lambda}{2} \right) - \frac{\lambda}{4}(1 - \lambda^2r_B) \right]$$

The argument is very similar for jurisdiction $W$ and obviously in both jurisdictions, the unanimity threshold for consolidation is always greater than the majority one. Note that, majority threshold is referred to the minimal public good cost that makes more than half of the population to vote for consolidation and unanimity threshold is the minimal $K$ which is necessary to get the votes of the whole population.

### 4 Joint vs. Independent Provision

Individuals vote for the options of public good provision taking into account the costs and benefits of each case. In this section we first examine the characteristics of the outcomes when the majority or unanimity approval is needed for consolidation. Then, we check under which conditions consolidation is a welfare improvement for the whole population.

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7 The majority threshold and its computation for jurisdiction $W$ are given in the appendix.

8 To avoid repetition, we do remaining analyses for only jurisdiction $B$ and also by taking account the analytical facility, only the unanimity voting system is followed through the paper. However, our conclusions are less strict but still robust for majority voting system.
4.1 Conditions

In section 3, we have found the necessary thresholds to get the approval of each jurisdiction. However, for consolidation the approval of both sides is necessary. To characterize the necessary conditions for consolidation, first remember from Proposition 1, that jurisdiction $B$ is unanimously in favor of consolidation if and only if

$$K > \frac{\lambda}{1-\lambda} \left[ 1 - \lambda^{r_B} \left( \frac{1}{2} + \frac{\lambda}{2} \right) \right] \equiv K_B (\lambda, r_B) \quad (3)$$

Moreover, we know that all individuals in jurisdiction $W$ will be in favor of consolidation, if the median voter $w_m = (1 + \lambda)/2$ is in favor of it. Then, the unanimity threshold for jurisdiction $W$ is

$$K > \frac{1-\lambda}{\lambda} \left[ 1 - (1 - \lambda)^{rw} \left( 1 - \frac{\lambda}{2} \right) \right] \equiv K_W (\lambda, r_W) \quad (4)$$

It is possible to find a value $K_H$ that assures the approval of both jurisdictions. There also exists a critical cost value $K_L$, such that if the public good cost is lower than that critical value, at least one of the jurisdictions does not approve the consolidation.

**Proposition 3** Independently of the jurisdiction sizes, if the cost of the public good is high enough, $K > K_H$, both communities are in favor of consolidation and if it is below a threshold, $K < K_L$, consolidation does not occur. If the public good cost is between these two thresholds, $K_H > K > K_L$, the consolidation is approved by both jurisdictions only for some critical values of $\lambda$ where $\lambda \in (\lambda_W (K, r_W), \lambda_B (K, r_B))$.

**Proof.** For consolidation to occur, both (3) and (4) should be satisfied. Now note that $\frac{\partial K_W (\lambda, r_W)}{\partial \lambda} < 0$, $\frac{\partial K_B (\lambda, r_B)}{\partial \lambda} > 0$. Thus for a given value of $r_W$, $K_W (\lambda, r_W)$ takes its maximum value for $\lambda = 0$. $\frac{\partial K_B (\lambda, r_B)}{\partial \lambda} > 0$, so for a given $r_B$ its maximum is at $\lambda = \frac{1}{2}$ (recall that $\lambda \in [0, \frac{1}{2}]$). Obviously, if $K > \max \{K_W (0, r_W), K_B (\frac{1}{2}, r_B)\}$, both conditions will be satisfied and consolidation will occur for all values of $\lambda$. Furthermore, the necessary condition for approval is an increasing function of $r$, i.e., if the importance of group composition increases, the consolidation is less likely to occur.

For consolidation not to occur at least one of the two approval conditions should not be satisfied. Note that $\frac{\partial K_W (\lambda, r_W)}{\partial \lambda} < 0$, $\frac{\partial K_B (\lambda, r_B)}{\partial \lambda} > 0$. Thus,

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$^9$See the Appendix for the derivation of this condition.
showing that $K_W(0, r_W) > K_B(0, r_B)$ and $K_B\left(\frac{1}{2}, r_W\right) > K_W\left(\frac{1}{2}, r_B\right)$, will be enough to assure a unique intersection, $\lambda^*$. Below this intersection value $K_W(\lambda^*) = K_B(\lambda^*) \equiv K_L$, at least one of the conditions is not satisfied.

We have $K_W(0, r_W) - K_B(0, r_B) = \frac{1}{2} + r_W$ and we know that $r_W$ is a positive parameter. It is trivial to obtain $K_B\left(\frac{1}{2}, r_W\right) - K_W\left(\frac{1}{2}, r_B\right)$ which is $\frac{3}{4}(0.5^{r_W} - 0.5^{r_B})$. If $r_B > r_W$, then $K_B\left(\frac{1}{2}\right) - K_W\left(\frac{1}{2}\right)$ is also positive. That is, if $K < K_B(\lambda^*) = K_W(\lambda^*)$, the consolidation will be rejected by at least one of the jurisdictions. Therefore, there will be one public good in each jurisdiction, independent of the jurisdictions' size. If $r_B > r_W$, then $K_W > K_B$ for any values of $\lambda$, which means if $K < K_W\left(\frac{1}{2}\right)$, the consolidation is not approved. That is also to say, only the threshold of the large jurisdiction is binding.

If $\max\{K_W(0), K_B\left(\frac{1}{2}\right)\} > K > K_W(\lambda^*)$, consolidation will be approved if and only if $\lambda \in (\lambda_W(K, r_W), \lambda_B(K, r_B))$ with $\frac{\partial \lambda_W(K, r_W)}{\partial K} < 0$, $\frac{\partial \lambda_B(K, r_B)}{\partial K} > 0$, i.e., for some values of public good cost, consolidation occurs if and only if the smaller jurisdiction has a moderate size. ■

Intuitively, if the public good cost is high enough, the benefits of consolidation are sufficient to compensate the loss caused by heterogeneity and both communities are in favor of consolidation independent of the size of the jurisdiction. If the public good cost is below a critical level, for any size of jurisdictions at least one of the jurisdictions would reject consolidation. Thus, in that case consolidation does not occur regardless of jurisdictions’ sizes.

We also observe that large jurisdictions are less likely to consolidate. If $r_B \leq r_W$, the approval threshold of jurisdiction $W$ is higher than the approval threshold of jurisdiction $B$ and the consolidation occurs if the small jurisdiction is sufficiently large. However, for $r_B > r_W$, when jurisdictions have similar size, jurisdiction $B$ is less likely to consolidate since its population is less and therefore it enjoys less the public good. Hence, when the small jurisdiction is more intolerant, consolidation occurs if and only if the smaller jurisdiction has a moderate size. This is a consequence of preferences over the beneficiaries’ composition. In case of consolidation, small sized jurisdiction members become minority, therefore, enjoy less the public good. Hence, they are less likely to consolidate if their size is relatively too small.

4.2 Welfare Comparison

We now check the conditions under which consolidation is a welfare improvement. A consolidation is an improvement if the total utility of society increases when jurisdictions consolidate. The total welfare under consolidation is
When they provide the public good independently, the total utility is

\[ U_c = \int_0^\lambda [y - t_i + \lambda^{\prime} u (1 - d_i)] \, di + \int_0^1 [y_i - t + (1 - \lambda)^{\prime} w (1 - d_i)] \, di \]

\[ = \lambda \left[ \lambda^{\prime} u \left( 1 - \frac{1 - \lambda}{2} \right) \right] + (1 - \lambda) \left[ (1 - \lambda)^{\prime} w \left( 1 - \frac{1}{4} \left( 1 - \lambda + \frac{\lambda^2}{1 - \lambda} \right) \right) \right] + y - K \]

When they provide the public good independently, the total utility is

\[ U_s = \int_0^\lambda [y - t_i + (1 - d_i)] \, di + \int_0^1 [y_i - t + (1 - d_i)] \, di \]

\[ = \lambda \left[ \left( 1 - \lambda \right) \frac{1}{4} + y - 2K \right] + (1 - \lambda) \left[ \left( 1 - \lambda \right) \frac{1}{4} + y - 2K \right] \]

By definition, consolidation is an improvement if \( U_c > U_s \). That is satisfied if and only if

\[ K > \frac{3}{4} + (1 - \lambda) \frac{\lambda}{2} - \lambda^{\prime} u + \lambda^{\prime} w \left( \frac{1 + \lambda}{2} \right) - (1 - \lambda)^{\prime} w \left( \frac{3 - 2 \lambda - 2 \lambda^2}{4} \right) = K_{imp}(\lambda) \]

We define \( K_{imp}(\lambda) \) as the improvement threshold which means that, if the cost of the public good is greater than this threshold, the consolidation is an improvement independent of the jurisdictions’ sizes. Note that \( \frac{\partial K_{imp}(\lambda)}{\partial \lambda} > 0 \). Hence, \( K_{imp}(\lambda) \) is maximized at \( \frac{1}{2} \). Then, if \( K > \frac{3}{8} - \frac{3}{8} \left( \frac{1}{2 w} + \frac{1}{2 w} \right) = K_{imp} \left( \frac{1}{2} \right) \), the consolidation is an improvement for all values of \( \lambda \). If \( K < K_{imp} \left( \frac{1}{2} \right) \), consolidation is an improvement for the society if and only if \( \lambda \in (0, \lambda_{imp}(K)) \) with \( \frac{\partial \lambda_{imp}(K)}{\partial K} > 0 \). That yields to the following proposition.

**Proposition 4** If the public good cost is greater than the "improvement" threshold, i.e. \( K > K_{imp} \), consolidation is an "improvement" for any size of jurisdictions. If the public good cost is not sufficiently high, consolidation is "improvement" if and only if small jurisdiction is small enough (\( \lambda < \lambda_{imp} \)).

In other words, the total utility of the whole society increases by consolidation, if the cost of the public good is high enough and if not, the small jurisdiction should be small enough compared to the large one.
5 Location decision

In this section, we analyze the mechanisms for the decision of public good location. First, we check under which conditions an agreement for consolidation can be obtained by changing the location of the public good. Then, we analyze the case where a social planner decides on the location of the public good with the purpose of maximizing the utility of the jurisdiction.

5.1 Mediation

From Proposition 3, we know the conditions for approval of consolidation by both jurisdictions. Those decisions are taken by residents that are aware of the second stage decision mechanism which is majority voting. Now, imagine that there is a third party that may change the public good location to get the approval for consolidation. First the location under the case of consolidation is announced, then individuals vote. Obviously, if no jurisdiction approves the consolidation for the median voter’s location, there is no way to get the approval by changing the location of the public good, i.e., mediation is worthless. That means, if only one of the jurisdictions is against consolidation, it may be possible to take the majority approval of the other jurisdiction by the help of a third party.

Let us assume that the intolerance factor is the same for both jurisdictions, \( r_W = r_B = r \). This assures us that if community \( W \) approves the consolidation, community \( B \) does as well. This also means that if the small jurisdiction does not approve the consolidation for the location \( \frac{1}{2} \), no agreement is possible. Then, the interesting case is where the consolidation is approved by jurisdiction \( B \) but not by jurisdiction \( W \). We should first find out under which conditions it is possible to convince the community \( W \) to consolidate by changing the location of the public good. We know that if the location of the public good under consolidation moves toward the median voter’s location of a community, individuals in that community become more likely to consolidate. Hence, the community \( W \) may be convinced to consolidate if and only if

\[
K > \frac{1 - \lambda - (1 - \lambda)^{r+1}}{\lambda}
\]

However, the above condition is not sufficient to approve the consolidation, since while convincing one jurisdiction, the other may become against to it. Jurisdiction \( B \) continues to prefer consolidation to independent provision if the public good cost is higher than community’s threshold for a given location, i.e., \( K > K_B(L) \) where \( L \) is the new location of the public good.

We know that since the benefits of economies of scale are higher for a smaller jurisdiction, smaller jurisdictions are more likely to accept the new location or approval of consolidation. Finally, we can observe that when \( \lambda \) is small, although it is harder to have consolidation, it is easier to convince the large jurisdiction to
approve it. In other words, disagreements over consolidation get harder to solve when jurisdictions have relatively similar sizes or when they are more intolerant about sharing a public good.

5.2 Social planner

Another alternative of majority voting mechanism in location decision is to leave that decision to a social planner. A social planner could maximize the total utility of the jurisdiction by replacing the public good. Different from the previous case, here we talk analyze the second stage where the social planner takes the decision over consolidation as given, i.e., jurisdictions go to a social planner after deciding to consolidate or not. If the jurisdictions provide the public good independently, the social planner decides to the location in jurisdiction \( B \) by maximizing

\[
\frac{\lambda}{\int_0^\lambda y_i - t_i + (1 - d_i(m)) \, di}
\]

That gives us \( m_{eff}^B = \frac{\lambda}{2} \), that is to say, the public good will be located at \( \frac{\lambda}{2} \). For jurisdiction \( W \), the planner maximizes

\[
\frac{1}{\lambda} \int_0^\lambda y_i - t_i + (1 - d_i(m)) \, di
\]

and gets the efficient location of the public as \( \frac{1 + \lambda}{2} \), i.e. \( m_{eff}^W = \frac{1 + \lambda}{2} \).

In both jurisdictions the efficient location of the public good is the same as the location that is decided by majority voting.

However if they consolidate, the social planner will be maximizing

\[
\frac{\lambda}{\int_0^\lambda y_i - t_i + \lambda r_B (1 - d_i(m)) \, di} + \frac{1}{\lambda} \int_{\lambda}^1 [y_i - t_i + (1 - \lambda) r_w (1 - d_i(m)) \, di]
\]

and by rearranging the first order conditions we obtain the efficient location \( m_{eff} \) of the public good.

**Proposition 5** In case of consolidation, the efficient location of the public good is given by

\[
m_{eff} = \frac{1}{2} + \frac{\lambda}{2} \left(1 - \frac{\lambda r_B}{(1 - \lambda) r_w}\right)
\]
First of all note that, if individuals do not care about the composition of beneficiaries i.e. if \( r = 0 \), the efficient location of the public good is the same as the one decided by majority voting. However, when \( r_B(r_W) \) increases, the location of public good gets farther from jurisdiction \( B(W) \), i.e., when a jurisdiction cares more about the composition of the beneficiaries, for efficiency, the public good will be located farther away from this jurisdiction, that is:

\[
\frac{\partial m_{eff}}{\partial r_B} > 0 \quad \frac{\partial m_{eff}}{\partial r_W} < 0
\]

Note also that when \((1 - \lambda) r_W > \lambda r_B\), the efficient location of the public good is on the right of the median voter of the society \( m_{eff} > \frac{1}{2} \). This also means that when the location of the public good is decided by a planner, the small jurisdiction will be more keen on separation than when it is decided by majority.

When jurisdictions independently provide the public good or if they do not care about the beneficiaries’ composition, the efficiency requirement is equivalent to the majority voting requirement, i.e., each jurisdiction places the facility at the location of its median resident. However, in case of consolidation, the efficient location of the public good depends on the importance of group composition for each type and size of jurisdictions.

When people equally care about the group composition, the efficient location of the public good is greater than \( \frac{1}{2} \), that means it favors the larger jurisdiction. Moreover, when members of one group care more about the composition of beneficiaries, the efficient location gets farther away from the median voter of that group. In other words, when the location of the public good is decided by a social planner, i) the strategic choice of a group should be to reduce the importance of beneficiaries’ composition in the group. ii) mostly the regions where the minority lives loses because of the existence of group composition importance.

6 Non-excludable public good

So far, we have concentrated on the case of an excludable public good. However, exclusion is not possible for some public goods such as parks, and if the public good is non-excludable, jurisdictions beside the joint or separate provision options have the free-riding option. Under specific conditions, communities may prefer free-riding to independent provision or consolidation. Moreover, even if the communities prefer to provide their own public good, there may be an individual or a group of individuals that admire the others’ public good and therefore would use that one.
6.1 Free-riding

If a jurisdiction chooses to free ride, it means that individuals in that jurisdiction do not pay any taxes but they have no rights on the decision of the public good location. Not surprisingly, free-riding is a matter for only small jurisdiction. In other words, if the small jurisdiction decides to provide its own public good, large jurisdiction does as well. When jurisdiction $B$ free rides, the location of the public good is determined by residents in jurisdiction $W$, so located at $\frac{1+\lambda}{2}$.

If the following condition is satisfied, the free riding option is preferred by unanimity to consolidation.

$$U_c = y - K + \lambda \tau^B (1 - (\frac{1}{2} - \frac{\lambda}{2})) < y + \lambda \tau^B (1 - (\frac{1}{2} + \frac{\lambda}{2} - \frac{\lambda}{2})) = U_{fr}$$

$$-K + \lambda \tau^B (\frac{1+\lambda}{2}) < \lambda \tau^B \frac{1}{2}$$

That is, if $K > \frac{\lambda \tau^B}{2} = K_1$, (i), they will not consolidate but free ride.

When jurisdictions have the possibility of independent provision of the public facility,

$$U_s = y - \frac{K}{\lambda} + 1 < y + \lambda \tau^B (1 - (\frac{1+\lambda}{2} - \frac{\lambda}{2})) = U_{fr}$$

rearranging it the inequality can be expressed

$$1 - \lambda \tau^B \frac{1}{2} < \frac{K}{\lambda}$$

That is, if $K > (1 - \lambda \tau^B) \lambda = K_2$, (ii), jurisdiction $B$ prefers free riding to providing the public good independently.

Note that when (ii) is satisfied, (i) is also satisfied, that is, $K_2 > K_1$. The small jurisdiction prefers independent provision of the public good if the public good cost is between those critical values, $K_2 > K > K_1$.

When $r_B$ increases, free riding against consolidation is more likely ($K_1$ decreases), but if independent provision is possible, free riding is less likely ($K_2$ increases) since they prefer to provide the public good independently. However, when individuals do not care about the composition ($r = 0$), both equations are equal $K_2 = K_1$, i.e. the choice of the minority against consolidation is not affected whether it has the independent provision option or not. In other words, if there exists no intolerance against the other group, the minority would consolidate or free ride.

The free riding option may also affect the location decision of jurisdiction $W$. For some values of the public good cost, the majority in community $W$ may prefer to locate the public good on the right side of the median voter’s location to make the small community to provide its own public good. That threat is not credibly without a commitment, since there will be no more incentives to change the location in the second stage. However, imagine that jurisdictions
take their decisions about location in the first stage as well. Instead of voting for the location in the second stage, they vote for a pack that includes both the choice about how to provide the public good and where to locate it. Or you can only imagine that the large jurisdiction decides the location of its own public good and after that the small jurisdiction decides to free ride or not. Those two cases create incentives for the large jurisdiction to change the location of its public good. If members of the community \( W \) do not care about with whom they share the public good \( (r_W = 0) \), the free riding decision does not affect their location decision, i.e. it is located at \( \frac{1+\lambda}{2} \). However, if individuals in \( W \) value the beneficiaries' composition \( (r_W > 0) \), and the small jurisdiction is likely to free ride, \( K > (1 - \frac{K_B}{2}) \lambda \), then the majority in the jurisdiction \( W \) chooses to locate the public good to a point \( x \) greater than \( \frac{1+\lambda}{2} \) where \( x \) is derived from \( \min\left[\frac{3+\lambda}{2} - (1 - \lambda)^{rw}, 1\right] \). The majority prefers strategically the point \( x \) if it makes the minority to provide their own public good instead of free-riding. More intuitively, in a democracy, the majority group in the population may move its own public good to a more extremist point as a result of the intolerance against minority.

6.2 Envy

When we have two non-excludable public goods, an individual or a group of individuals from a jurisdiction may prefer to use the one that is not in their jurisdiction.\(^{10}\) Different from free riding, each community provides a public good and each individual pays an amount of tax to provide the public good for his community but if the public good of the other jurisdiction is sufficiently close to an individual's ideal point, he may have an intention to use it.

It is easy to verify that when \( r_i > 0 \) with \( i = W, B \), a single individual never prefers to use the other jurisdiction’s public good, since the utility that he gets from the other public good will be equal to zero. We check under what conditions there exists a group that would prefer to use the other jurisdiction’s public good. First, we observe that in jurisdiction \( B \) there exists no group of individuals that would prefer to use the public good in jurisdiction \( W \) if they have provided one in their jurisdiction. However, this is not always the case for members of large jurisdiction. No group of individuals from jurisdiction \( W \) prefer to use the public good of the others if there is not enough differences in sizes of jurisdictions and/or if the members of community \( W \) is very intolerant against sharing the good with other community members, i.e., if \( \lambda \) and/or \( r_W \) is high enough.

\(^{10}\)In the literature, this fact is mostly studied for excludable public goods for stability purposes. Since in this paper we do not let any other coalition rather than the grand and segregated ones, we take a glance to that question for the non-excludable case.
7 Conclusion

Although the preferences over population composition is a highly accepted concept, there is very little attention on its effects over local public good provision. In this paper, we have studied the effects of the existence and the changes of preferences over beneficiaries’ composition on various decisions.

We first look at each jurisdiction’s specific decision over consolidation where the trade off is between economies of scale and heterogeneity. However, different from the existing literature, this paper analyzes the effects and results of preferences over the composition of beneficiaries while deciding for consolidation in the provision of a public good. If the residents of a jurisdiction loose more utility sharing the public good with the other jurisdiction, the consolidation is less likely and we find that small jurisdictions are more keen to consolidation. In this framework, depending on the values of the public good cost and intolerance factor, the consolidation is more likely to occur when the small jurisdiction has a moderate or similar size compared to the large jurisdiction. Similar to the existing literature, we find that the consolidation decision increases the total utility of the society if and only if the small jurisdiction is small enough.

We also point out the effects of location decision and free-riding option. We obtain that it is harder to solve the disagreement when jurisdictions have similar sizes. Moreover, when a jurisdiction cares more about the composition of the public goods’ beneficiaries, the social planner’s location decision for the public good changes in a way that makes that jurisdiction worse off. Finally, the minority is more keen to provide their own public good even though they have the free-riding option and unlike the large community members, none of the small jurisdiction members would prefer to use the others’ public good if they have their own.

The current study may be developed in several directions such as analyzing the effects of different types of distributions and voting systems. Moreover, giving a strategical choice like lobbying may give interesting results. Lastly, it may be fruitful conducting an empirical analysis that looks simultaneously on the effects of heterogeneity in the decision of consolidation for the provision of a public good.
References


Appendix

Thresholds of Jurisdiction \( W \):

i) For unanimity approval in jurisdiction \( W \), from Proposition 1 we know that the median voter of jurisdiction \( W \) should be in favor of the joint provision. Thus, the below condition should be satisfied:

\[
y - K + (1 - \lambda)^{rw} \left( 1 - \frac{1}{2} - \frac{1 + \lambda}{2} \right) > y - \frac{K}{1 - \lambda} + 1
\]

by rearranging terms, we obtain the unanimity threshold for the jurisdiction \( W \):

\[
K > \frac{1 - \lambda}{\lambda} \left[ 1 - (1 - \lambda)^{rw} \left( \frac{1 + \lambda}{2} \right) \right] = K_W (\lambda)
\]

ii) For majority approval in jurisdiction \( W \); note that the individual located at \( \frac{1 + \lambda}{2} - x \) where \( x < \frac{1}{2} \) prefers consolidation if and only if

\[
K > \frac{1 - \lambda}{\lambda} \left[ 1 - (1 - \lambda)^{rw} \left( \frac{1 - \lambda}{2} \right) - x(1 + (1 - \lambda)^{rw}) \right] = K_{\frac{1 + \lambda}{2} - x}
\]

and an individual at \( \frac{1 + \lambda}{2} + \bar{x} \) approves consolidation if and only if

\[
K > \frac{1 - \lambda}{\lambda} \left[ 1 - (1 - \lambda)^{rw} \left( \frac{1 - \lambda}{2} \right) - \bar{x}(1 - (1 - \lambda)^{rw}) \right] = K_{\frac{1 + \lambda}{2} + \bar{x}}
\]

Notice that \( K_{\frac{1 + \lambda}{2} - x} \) and \( K_{\frac{1 + \lambda}{2} + \bar{x}} \) are decreasing in \( x \) and \( \bar{x} \) respectively. Thus, for values of \( K \) where \( K > K_{\frac{1 + \lambda}{2} - x} = K_{\frac{1 + \lambda}{2} + \bar{x}} \) for \( \bar{x} + x = \frac{1 + \lambda}{2} \), at least the half of the population in jurisdiction \( W \) is in favor of consolidation, that is, consolidation is approved by majority. When \( K = K_{\frac{1 + \lambda}{2} - x} = K_{\frac{1 + \lambda}{2} + \bar{x}} \) and \( \bar{x} + x = \frac{1 + \lambda}{2} \) exactly the half of the jurisdiction would be in favor of consolidation.

From \( K_{\frac{1 + \lambda}{2} - x} = K_{\frac{1 + \lambda}{2} + \bar{x}} \) and \( \bar{x} + x = \frac{1 + \lambda}{2} \) we have \( \bar{x} = \frac{1 + \lambda}{4} (1 + (1 - \lambda)^{rw}) \) and \( x = \frac{1 + \lambda}{4} (1 - (1 - \lambda)^{rw}) \).

So, if \( K > \frac{1 + \lambda}{4} \left[ 1 - (1 - \lambda)^{rw} \left( 1 - \frac{1}{2} \right) - \frac{1 + \lambda}{4} (1 - (1 - \lambda)^{rw}) (1 + (1 - \lambda)^{rw}) \right] \)

\[\iff K > \frac{1 + \lambda}{4} \left[ 1 - (1 - \lambda)^{rw} \left( \frac{1}{2} \right) - \frac{1 + \lambda}{4} (1 - (1 - \lambda)^{2rw}) \right] \]

the majority of jurisdiction \( W \) is in favor of consolidation.

Note that if \( x = \frac{1 + \lambda}{4} (1 - (1 - \lambda)^{rw}) > \frac{1}{2} \), that is, \( \frac{1 + \lambda}{2} - x < \frac{1}{2} \), the condition \( K_{\frac{1 + \lambda}{2} - x} \) slightly changes. In that case,

\[
K > \frac{1 - \lambda}{\lambda} \left[ 1 - (1 - \lambda)^{rw} \left( \frac{1}{2} + \frac{\lambda}{2} \right) - x(1 - (1 - \lambda)^{rw}) \right] = K_{\frac{1 + \lambda}{2} - x}
\]
Notice that \(x = \frac{1 - \lambda}{4} (1 - (1 - \lambda)^{rw}) > \frac{1}{2}\) if and only if \((1 - \lambda)^{rw+1} + 3\lambda < 1\).

Now, for majority approval we need to have \(\bar{x} + x = \frac{1 - \lambda}{2}\), and this is satisfied for \(x = \frac{1}{4} - \frac{\lambda}{4} \left( \frac{1+(1-\lambda)^{rw}}{1-(1-\lambda)^{rw}} \right)\) and \(\bar{x} = \frac{1}{2} - \frac{\lambda}{2} + \frac{\lambda}{4} \left( \frac{1+(1-\lambda)^{rw}}{1-(1-\lambda)^{rw}} \right)\). Then, the below condition should be satisfied.

\[
K > \frac{1 - \lambda}{\lambda} \left[ 1 - (1 - \lambda)^{rw} \left( 1 - \frac{\lambda}{2} \right) - \left[ \frac{1}{4} - \frac{\lambda}{2} + \frac{\lambda}{4} \left( \frac{1+(1-\lambda)^{rw}}{1-(1-\lambda)^{rw}} \right) \right] (1 - (1 - \lambda)^{rw}) \right]
\]

which can be written as

\[
K > \frac{1 - \lambda}{\lambda} \left[ 1 - (1 - \lambda)^{rw} \left( 1 - \frac{\lambda}{2} \right) - \left[ \frac{1}{4} - \frac{3\lambda}{4} - \frac{\lambda}{2} \left( \frac{1}{(1-\lambda)^{rw}} - 1 \right) \right] (1 - (1 - \lambda)^{rw}) \right]
\]

after some steps of simplification the condition for majority approval of jurisdiction \(W\) when \((1 - \lambda)^{rw+1} + 3\lambda < 1\) is

\[
K > \frac{1 - \lambda}{\lambda} \left[ \frac{3 + \lambda}{4} - \frac{3 + \lambda}{4} (1 - (1 - \lambda)^{rw}) \right]
\]

Majority approval of free-riding decision:

When we are looking for majority of votes to approve the free riding decision, it is easy to show that the essence of the result does not change. Obviously, the small jurisdiction is more keen to free ride.

If majority approval is needed for free riding \(K > \left( 1 - \frac{\lambda^B}{2} - \frac{\lambda}{2} (1 - \lambda^B) \right) \lambda = K_{m2}\) and \(K_{m1} = K_{1}\).

\(K_{m2} > K_{m1}\) since \(1 > \lambda^B + \frac{\lambda}{4} (1 - \lambda^B)\).