MEASURING VALUE AT RISK OF PORTFOLIOS UNDER THE EDGEWORTH-SARGAN DISTRIBUTION.

by

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Abstract:

This paper sheds light on the evaluation of portfolio risk by assuming a distribution capable of incorporating the behaviour of most financial variables, especially at the tails: the so called Edgeworth-Sargan distribution. This density is preferable over other distributions, such as the Student's t , when fitting high frequency financial variables, because of its flexibility for improving data fits by adding more parameters in a natural way.

Furthermore, this distribution is easy to be generalised to a multivariate context and, therefore, correlation coefficients among variables can be estimated efficiently. This article thus provides new insights into VaR methodology by estimating the joint density of portfolio variables, and by calculating the right critical values of the underlying portfolio density as well. The empirical examples include the estimation and evaluation of different volatility and weight scenarios for portfolios composed of stock indices and interest rates for major financial markets.

Keywords: Value at Risk, portfolio choice, joint distribution, Edgeworth-Sargan distribution, GARCH models, optimal weighting.

JEL classification code: G10, G11, C13.

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1. Introduction¹.

Risk Management has become a topic of major interest for both researchers and practitioners in the last decades. Several theories have contributed to this development, such as the portfolio selection models (Markovitz, 1952), the capital asset pricing model (Sharpe, 1964; Lintner, 1965; Mossin, 1966) or the option pricing theory (Black and Scholes, 1973; Merton, 1973), among others. More recently, Value at Risk, or VaR, has experienced a wide development guided by the regulatory requirements for bank solvency (Group of Thirty, 1993, Basle Committee on Banking Supervision, 1995, 1996a,b and 2001). All these theories represent an overwhelming improvement in quantifying the risk of financial assets, but they still present several shortcomings basically owing to the normality assumption. The non-normality of high frequency financial variables has been widely tested in financial literature (Mandelbrot, 1963; Harvey and Zhoe, 1993; Dacorogna et al., 1995) finding that the density underlying most financial data is more peaked and heavy tailed than the normal distribution.

Many authors have tackled the non-normality of high frequency financial variables from different perspectives ranging from non-parametric (Gallant and Tauchen, 1989; Robinson, 1995; Newley et al., 1998) to parametric estimation of the underlying density, assuming other specifications like the Student's t (Praetz, 1972; Blattberg and Gonedes, 1974; Rogalski and Vinso, 1978), jump processes (Ball and Torous, 1983; Jorion, 1988), mixtures of normal distributions (Hamilton, 1991; Peiró, 1995), or many other densities (McDonald and Newley, 1988; Baille and Bollerslev, 1990; Mittnik and Rachev, 1993; McDonald and Xu, 1995). This literature also accounts for the conditional heteroskedasticity phenomenon inherent in this kind of data (Bollerslev, 1987; Baillie and Bollerslev, 1989; Hsieh, 1989; Nelson, 1991; Ding et al., 1993; León and Mora, 1999), the ARCH and GARCH processes by Engle (1982) and Bollerslev (1986) being the most widely used. However, only few papers focus on the theoretical implications of the non-normality of these variables in financial markets equilibrium

¹ Both authors acknowledge the financial support of the Digcyt under project SEC98-1112, the J.C. y L. under project SA29/99. Responsibility for the contents, as well as possible deficiencies, remains solely with the authors.

-Jurczenko and Maillet (2001) introduce a generalization of CAPM to three moment asset pricing model.

Regarding VaR literature, many different methodologies have been employed to approach to the underlying density, such as historical simulation data and Monte Carlo experiments or parametric estimation (Jorion, 1997; Coronado, 2000). The most popular method derived by J. P. Morgan Bank (1995) and implemented in “RiskMetrics” is based on the estimation of the variance and covariance matrix of portfolio assets. Therefore, it relies on the simplified, but probably unreasonable, normality hypothesis. The validity of this hypothesis on calculating VaR has been studied by Finger (1987) and Hull and White (1998), and some alternative specifications have been proposed recently: mixtures of normal distributions (Venkataraman, 1997), Student’s t (Vlaar, 2000; Lucas, 2000), Hyperbolic distributions (Bauer, 2000), switching regime models (Billio and Pellizon, 2000) or semi-parametric techniques (Danielsson and de Vries, 2000).

This article fits into this framework by introducing a new methodology for computing VaR which attempts to solve some of the deficiencies of previous approaches. This paper focuses on the assumption of the Edgeworth-Sargan distribution – hereafter ES – rather than the normal, which, in fact, is nested on the ES. This latter distribution has been shown capable of capturing salient empirical regularities of the histogram for most high frequency financial variables in Perote (1999) and Mauleón and Perote (2000). In particular, this distribution is able to account for thicker tails than the normal, as well as for possible asymmetries, as a result of the consideration of a general and flexible parameterisation. Therefore, the use of this density can be interpreted as a semi-non-parametric approach to the underlying density, moreover, the parameter flexibility makes data fits improve, compared to other fitted densities – like Student’s t , for instance. Moreover, the ES distribution can be generalised to a multivariate context (Mauleón and Perote, 1999) and thus the variance and covariance matrix of a group of variables can be estimated consistently with the given assumptions. The analytical simplicity of ES

represents another advantage of this distribution, since it does not only make optimisation algorithms converge, but also the estimated distribution can be used to calculate probabilities or critical values for confidence intervals, as shown in Perote and Del Brío (2001).

The next section (2) outlines the general model considered for calculating a portfolio's VaR, and section 3 describes in detail the particular case analysed in the empirical examples. The empirical results (section 4) are drawn from a sample of daily observations, spanning approximately 14 years of stock indices and long run interest rates for major financial markets. Finally, conclusions are gathered in section 5.

2. General portfolio model.

The VaR of some portfolio describes the worst expected loss of the portfolio in some temporal horizon and at some confidence level. Hence the computation of VaR involves dealing with four main elements: time horizon, confidence level, underlying distribution and portfolio time varying covariances. The time horizon and the choice of the confidence level depend on the risk aversion of the portfolio manager. For example, RiskMetrics recommends the Daily Earning at Risk – hereafter DEaR – whilst Basle Committee shows preference for the ten days period. However, the decision is not really relevant unless structural changes altered the conditional density. Regarding the confidence level, it is clear that the bigger the risk aversion the smaller the confidence level to be used.

The other two elements are equivalent under the commonly assumed normality assumption, since the whole structure of the multivariate normal only depends on their variance and covariance matrix¹. Nevertheless, other distributions, such as the ES, depend on more parameters to account for higher order moments. That is probably the main contribution of this paper: the comparison of normality assumption (case A) versus a more general and complex structure (case B) that incorporates further moments which affect the computation of a portfolio's VaR.

As far as the time varying variance and covariance matrix are concerned, many assumptions may be considered, the ARCH and GARCH models by Engle (1986) and Bollerslev (1987) being the most widespread. In fact, RiskMetrics uses a GARCH(1,1) structure subject to some restricted hypotheses to guarantee the non-stationarity of conditional variances. Our approach does not assume unrestricted GARCH(1,1) processes but constant correlation coefficients², which help the optimisation algorithms to achieve positive definite matrices. Nevertheless, no other simplifying hypotheses are imposed to the variance and covariance matrix³. All of these factors are considered in the following portfolio model.

Let us define the return of some portfolio at time t (P_t) as a linear combination⁴ of the returns obtained by their N assets (r_{it}):

$$P_t = \sum_{i=1}^N w_{i,t-1} r_{it}, \text{ where } 0 \leq w_{i,t-1} \leq 1 \ \forall i = 1, 2, \dots, N \text{ and } \sum_{i=1}^N w_{i,t-1} = 1. \quad (2.1)$$

Without loss of generality, the returns on all assets are assumed to have zero mean and conditional variance at time t which is proportional⁵ to a GARCH(1,1) structure:

$$E[r_{it}] = 0 \text{ and } E[r_{it}^2] = \sigma_{it}^2 = c_i k_{it}^2, \ c_i > 0 \text{ and } k_{it}^2 = \alpha_{i0} + \alpha_{i1} k_{it-1}^2 + \alpha_{i2} r_{it}^2, \text{ where} \\ \alpha_{i0} > 0, \ 0 \leq \alpha_{is} \leq 1 \ \forall s = 1, 2 \text{ and } \alpha_{i1} + \alpha_{i2} < 1, \ \forall i = 1, 2, \dots, N. \quad (2.2)$$

Let also assume that the correlation of returns on assets i and j at time t can be written as follows:

$$\sigma_{ijt} = \rho_{ij} \sigma_{it} \sigma_{jt} \text{ where } 0 \leq \rho_{ij} \leq 1 \ \forall i = 1, 2, \dots, N \text{ and } \forall j = 1, 2, \dots, N. \quad (2.3)$$

If it is also assumed that returns on all assets are normally distributed (case A), the distribution of the portfolio is fully characterised, since normal distribution depends only on their first and second moments and linear combinations of normal variables are also normally distributed.

Case A:

Let us suppose that $r_{it} \sim N(0, \sigma_{it}^2)$ and let $g_i(r_{it})$ be its pdf, for all i. Note that these variables are not independent and thus their joint behaviour can be summarised as follows:

$$R_t \sim N(0, \Sigma_t) \quad (2.4)$$

$$\text{where } R_t' = [r_{1t} \quad r_{2t} \quad \Lambda \quad r_{Nt}] \text{ and } \Sigma_t = \begin{bmatrix} \sigma_{1t}^2 & \sigma_{12t} & \Lambda & \sigma_{1Nt} \\ \sigma_{12t} & \sigma_{2t}^2 & \Lambda & \sigma_{2Nt} \\ M & M & M & M \\ \sigma_{1Nt} & \sigma_{2Nt} & \Lambda & \sigma_{Nt}^2 \end{bmatrix}, \quad (2.5)$$

(Σ_t being a positive definite matrix).

Under these assumptions, the portfolio is distributed according to:

$$P_t \sim N(0, \sigma_{pt}^2) \quad (2.6)$$

$$\text{where } \sigma_{pt}^2 = W_t' \Sigma_t W_t = \sum_{i=1}^N w_{it}^2 \sigma_{it}^2 + \sum_{i=1}^N \sum_{j=1, j \neq i}^N w_i w_j \sigma_{ijt} = \sum_{i=1}^N w_{it}^2 \sigma_{it}^2 + \sum_{i=1}^N \sum_{j=1, j \neq i}^N w_i w_j \rho_{ij} \sigma_{it} \sigma_{jt}$$

$$\text{and } W_t' = [w_{1t} \quad w_{2t} \quad \Lambda \quad w_{Nt}] \quad (2.7)$$

This previous model is basically the one assumed by RiskMetrics and most risk dealers. Nevertheless, a straightforward generalisation is possible by considering further order moments also affecting VaR. That is defined in case B.

Case B:

Let r_{it} be distributed as Edgeworth-Sargan, hereafter⁶ $r_{it} \sim ES(0, \sigma_{it}^2)$. Therefore, densities, for each i, can be expressed in terms of 2.8.

$$f_i(r_{it}) = g_i(r_{it}) \{1 + q_i(r_{it})\} \quad (2.8)$$

where $g_i(r_{it})$ stands for the $N(0, k_{it}^2)$ pdf (k_{it}^2 defined in 2.2) and $q_i(r_{it}) = \sum_{s=2}^{S_i} d_{si} H_s(v_{it})$

represents a linear combination of the hermit polynomials $H_s(v_{it})$, d_{si} being the parameter of

each variable $v_{it} = \frac{r_{it}}{k_{it}}$ and for each polynomial s. These hermit polynomials fulfil the

following identity:

$$\frac{d^j g(v_{it})}{dv_{it}^j} = (-1)^j g(v_{it}) H_s(v_{it}) \quad (2.9)$$

where $g(v_{it})$ stands for a standard normal density, i.e. $N(0,1)$.

Following Kendall and Stuart (1977) hermit polynomials can be obtained⁷ by 2.10.

$$H_j(v_{it}) = v_{it}^j - \frac{j(j-1)}{2} v_{it}^{j-2} + \frac{j(j-1)(j-2)(j-3)}{2^2 2!} v_{it}^{j-4} - \frac{j(j-1)(j-2)(j-3)(j-4)(j-5)}{2^3 3!} v_{it}^{j-6} + \Lambda \quad (2.10)$$

Regarding the joint behaviour of portfolio variables (see 2.5), the Edgeworth-Sargan distribution can be also generalised to a multivariate context according to Mauleón and Perote (1999). These authors provided an expression for multivariate Edgeworth-Sargan distribution which satisfies an interesting property: their marginal distributions behave also as Edgeworth-Sargan. Therefore, under case B assumptions, we consider that

$$R_t \sim ES(0, \Sigma_t) \quad (2.11)$$

and the joint density of the vector R_t holds the following distribution:

$$F(R_t) = G(R_t) + \left[\prod_{i=1}^N g_i(r_{it}) \right] \left[\sum_{i=1}^N q_i(r_{it}) \right]. \quad (2.12)$$

where $G(R_t)$ stands for the multivariate normal density shown in 2.13.

$$G(R_t) = (2\pi)^{-N/2} |\Sigma_t|^{-1/2} \exp \left\{ -\frac{1}{2} R_t' \Sigma_t^{-1} R_t \right\} \quad (2.13)$$

It is easy to check that linear combinations of Edgeworth-Sargan distributions are also Edgeworth-Sargan distributed, and thus the portfolio holds:

$$P_t \sim ES(0, \sigma_{pt}^2) \quad (2.14)$$

where σ_{pt}^2 satisfies 2.7, although in this case it is worth noting that $\sigma_{it}^2 = (1 + 2d_{i2})k_{it}^2$.

3. Calculating VaR of a simple portfolio.

For the sake of clarity, this section is devoted to describe the kind and composition of the portfolios employed for empirical analysis in next section, i.e. two-asset portfolios. Therefore,

let P_t be a linear convex combination of r_{1t} , which might represents some stock market index, and r_{2t} , for example, treasury bond yields. Hence,

$$P_t = w_{1,t-1}r_{1t} + w_{2,t-1}r_{2t} \quad (3.1)$$

where $0 \leq w_{1,t-1} \leq 1$, $\forall i = 1, 2$ and $w_{1,t-1} + w_{2,t-1} = 1$.

Let us also assume that both assets have zero mean (both variables have previously been centred) and present conditional heteroskedasticity that can be explained reasonably well by some GARCH(1,1) process. Moreover, both assets are assumed to be correlated according to the coefficient ρ_{ij} . Therefore, the first and second moments of the variables may be expressed as follows:

$$E[r_{it}] = 0, E[r_{it}^2] = \sigma_{it}^2 = c_i k_{it}^2 = c_i (\alpha_{i0} + \alpha_{i1} k_{it-1}^2 + \alpha_{i2} r_{it}^2) \text{ and } E[r_{1t} r_{2t}] = \sigma_{12t} = \rho_{12} \sigma_{1t} \sigma_{2t}$$

$$c_i > 0, \alpha_{0i} > 0, 0 \leq \alpha_{si} \leq 1, \alpha_{1i} + \alpha_{2i} < 1 \text{ and } 0 \leq \rho_{12} \leq 1 \quad \forall s = 1, 2 \text{ and } \forall i = 1, 2.$$

(3.2)

Nevertheless, for further analysis of portfolio risk, it is necessary to assume some distributional hypotheses. Once more, case A reflects the commonly used normality assumption, whilst case B imposes a more complex structure based on the Edgeworth-Sargan distribution.

Case A:

The joint distribution of r_{1t} and r_{2t} is given in 3.3, and the distribution of the portfolio is derived in 3.4.

$$G(r_{1t}, r_{2t}) = \frac{1}{2\pi} \frac{1}{k_{1t} k_{2t} \sqrt{1 - \rho_{12}^2}} \exp \left\{ -\frac{1}{2} \frac{1}{(1 - \rho_{12}^2)} \left[\frac{r_{1t}^2}{k_{1t}^2} + \frac{r_{2t}^2}{k_{2t}^2} - 2\rho_{12} \frac{r_{1t} r_{2t}}{k_{1t} k_{2t}} \right] \right\}$$

(3.3)

$$g_p(P_t) = \frac{1}{\sigma_{pt} \sqrt{2\pi}} \exp \left\{ -\frac{1}{2} \frac{P_t^2}{\sigma_{pt}^2} \right\} \quad (3.4)$$

Note that k_{1t}^2 and k_{2t}^2 are defined in 3.2, P_t is described in 3.1, and σ_{pt}^2 satisfies 3.5.

$$\sigma_{pt}^2 = w_{1,t-1}^2 k_{1t}^2 + w_{2,t-1}^2 k_{2t}^2 + 2w_{1,t-1} w_{2,t-1} \rho_{12} k_{1t} k_{2t} \quad (3.5)$$

Notice that if all the parameters of the specification above are known ($w_{i,t-1}$, α_{i0} , α_{i1} , α_{i2} and ρ_{12} $\forall i = 1, 2$), the VaR of the portfolio for next period at confidence level α may be easily computed as indicated in 3.6.

$$VaR = \sigma_{pt} \phi(\alpha) \text{ where } \alpha = Prob[x \leq \phi(\alpha)] \text{ if } x \sim N(0,1). \quad (3.6)$$

Case B:

The joint distribution of r_{1t} and r_{2t} , and the portfolio density are shown in 3.7 and 3.8, respectively.

$$F(r_{1t}, r_{2t}) = G(r_{1t}, r_{2t}) + g_1(r_{1t}) g_2(r_{2t}) \{q_1(r_{1t}) + q_2(r_{2t})\} \quad (3.7)$$

$$\text{where } g_1(r_{1t}) = \frac{1}{k_{1t} \sqrt{2\pi}} \exp\left\{-\frac{1}{2} \frac{r_{1t}^2}{k_{1t}^2}\right\}, \quad g_2(r_{2t}) = \frac{1}{k_{2t} \sqrt{2\pi}} \exp\left\{-\frac{1}{2} \frac{r_{2t}^2}{k_{2t}^2}\right\},$$

$$q_1(r_{1t}) = \sum_{s=2}^{S_1} d_{1s} H_s(r_{1t}/k_{1t}) \text{ and } q_2(r_{2t}) = \sum_{s=2}^{S_2} d_{2s} H_s(r_{2t}/k_{2t}).$$

$$f_p(P_t) = g_p(P_t) \left\{1 + \sum_{s=2}^{S_p} d_{ps} H_p(P_t/\sigma_{pt})\right\} \quad (3.8)$$

It must be noticed that k_{1t}^2 and k_{2t}^2 are defined in 3.2, P_t is described in 3.1, σ_{pt}^2 satisfies 3.5, and $g_p(P_t)$ is stated in 3.4. Moreover, if all parameters of this model ($w_{i,t-1}$, α_{i0} , α_{i1} , α_{i2} , ρ_{12} and d_{ps} $\forall i = 1, 2$ and $\forall s = 2, \dots, S$) are known, the VaR of the portfolio for next period and at confidence level α may be obtained from 3.6, assuming that $\phi(\alpha)$ stands for the critical value of a standard Edgeworth-Sargan density, i.e. $x \sim ES(0,1)$. In fact, this is the main advantage of this approach, since the Edgeworth-Sargan distribution is able to account for the thick tails of high frequency financial data and, therefore, to measure the probability of extreme values more accurately. The only problem seems to lie in calculating these critical values, however, it does not represent a problem any longer, since the Edgeworth-Sargan distribution is

easy to integrate. Actually, if $x \sim ES(0,1)$ with pdf $f(\cdot)$ and $g(\cdot)$ stands for the $N(0,1)$ density, it is straightforward to prove that

$$Prob[x \leq \phi(\alpha)] = \int_{-\infty}^{\phi(\alpha)} f(x)dx = \int_{-\infty}^{\phi(\alpha)} g(x)dx - g(\phi(\alpha)) \sum_{s=2}^{S_p} d_s H_s(\phi(\alpha)).$$

4. Empirical results.

This section compares different portfolios' VaRs and different volatility scenarios. In particular, the models shown in previous section are estimated by maximum likelihood and the expected loss for next period – DEaR approach is assumed – is measured as a percentage of the investment and at different confidence level. For the sake of comparison, both the multivariate normal and the ES are estimated. Notice that under normal distribution, VaR can be computed directly through the estimated time varying variance and covariance matrix but, on the other hand, if the ES distribution is assumed the rest of the parameters of the portfolio distribution must be estimated before VaR is calculated, and thus the exact critical value.

Therefore, the methodology used in this section for calculating a portfolio VaR can be summarised in the following steps: firstly, the conditional multivariate distribution for portfolio variables is jointly estimated (either under normality or ES assumption). Secondly, the conditional ES density of the portfolio is estimated, subject to the estimates of the variance and covariance matrix obtained in the first step. Finally, the accurate critical value of the estimated ES for the portfolio – at each confidence level – is computed. A portfolio VaR is just the product of this critical value and the estimated conditional deviation for the portfolio. VaRs under both specifications are compared for different confidence levels (0.05, 0.025 and 0.01). As a consequence of the ES tails being thicker than those of the normal distribution, the bigger the confidence level, the bigger the expected difference in VaR between both approaches.

Tables 1 to 5 show the VaR estimates for different portfolios and scenarios. In particular, table 1 and 4 display the estimates for some portfolios composed of the Dow Jones index and long run interest rates in United States using different weights – 0.1, 0.5 and 0.9 –

and different time periods (table 1 intends to capture a low volatility period, whilst table 4 accounts for the higher volatile scenario after October 1987 stock market crash). Table 2 summarizes the results for some portfolios composed of the Footsie index and long run interest rates for United Kingdom and, finally, tables 3 and 5 show the VaR of some portfolios weighting Dow Jones and Footsie indices in different volatility scenarios. For the sake of clarity, parameter notation follows that of previous section.

It is worth to note that some of the parameters of the ES distribution have been constrained to zero after testing these hypotheses. In particular, no evidence of asymmetries were found (contrary to other authors like Theodossiou, 1998), and therefore all the odd parameters of the ES distribution were cancelled out. Moreover, the parameters of some densities were also eliminated to help optimisation algorithms to converge (especially in high volatility periods, i.e. tables 4 and 5). Notice that the advantages of the general model presented in previous section might vanish as the dimension of the portfolio increases, since the parametrical complexity of the model makes it empirically intractable. Nevertheless, this shortcoming only affects the estimation of the variance and covariance matrix jointly with the rest of the parameters of the multivariate ES distribution⁸. However, computational problems can be solve by selecting adequately the initial values for the optimisation algorithms.

The concrete daily series used in empirical work are the Dow Jones industrial price index (1/1/70 – 28/5/96), the FTSE total non-financial price index (1/1/70 – 28/5/96), the UK treasury 10 year benchmark bond yield (4/1/83 – 28/5/96) and the 10 year benchmark bond yield (1/1/90 – 28/5/96). The high volatility periods end the 13/11/87, just before October 1987 stock market crash. It must be also noted that an AR(1) process has previously been estimated for the daily variation rates of each variable and the residuals of these estimation have also been modelled⁹. Additionally, in order to make optimisation algorithms converge, a few outliers have been interpolated.

Turning to the empirical results shown in the tables, an asterisk indicates the non significant parameters at 5 percent confidence level. The log likelihood value of each density is

also displayed, they reflect the fact that the multivariate conditional ES distribution is clearly preferred to the multivariate conditional normal to represent the joint behaviour of portfolio variables. This evidence reinforces the evidence of the non-normality of these variables¹⁰ and, therefore, the more efficient estimation of portfolio density obtained by the more general ES distribution. Once this hypothesis is tested, a sensitivity analysis is performed, showing how VaR is influenced by the wrongly normality assumption for different portfolios, confidence levels and volatility scenarios.

The results clearly conclude that there exist both an overestimation and an underestimation of the true VaR when accounting only for the first and second moments. This evidence was also found by Lucas (2000) when comparing usual VaRs to these obtained under a Student's *t* distribution¹¹. Obviously, the bigger the confidence level the bigger the VaR understatement. Actually, only little overestimation is found for some portfolios at 5 percent confidence level (VaR under normal distribution is as much a 3 percent lower than under ES) and, on the other hand, VaRs assuming a normal density are, in most cases, sharply understated compared to those assuming an ES distribution (up to a 22 percent understatement is detected in some portfolios at 1 percent confidence level). This result is also in line with Danielsson and de Vries (2000), who assessed that at 5 percent confidence level RiskMetrics is adequate but underpredicts VaR compared to a semi-parametric method,¹² which is more accurate capturing portfolios' tails behaviour.

With respect to the factors or scenarios that explain the bigger understatements, it is not easy to adventure general conclusions, apart from the fact that underestimates increase as confidence levels decrease. Apparently, it is also clear that the bigger portfolio variance the bigger portfolio VaR, but this insight may be misleading, since one of the advantages of using an ES distribution consists on separating the variance parameter (accounting for dispersion in mean neighbourhood) from the rest of the parameters (capturing kurtosis or dispersion at the tails). For example, positive values for d_4 imply more kurtosis and, therefore, thicker tails than the normal distribution. Another effect, raised by Perote and Del Brío (2001), is the fact that the

ES distribution advantages on measuring risk appeared more clearly in high volatility scenarios, since it is more flexible to incorporate behaviours far from the mean by appending additional parameters. This evidence is highlighted by the increasing differences of VaR obtained in tables 1 and 3 compared to tables 4 and 5, respectively (especially for more volatile portfolios).

Another interesting outcome of this approach is the fact that the GARCH parameters are near non-stationarity, which is consistent with RiskMetriks methodology. As is well known, this persistence of variance captures part of the unconditional kurtosis (Engle and Bollerslev, 1986) and, thus, VaR differences are not as large as those that would be found if unconditional normal and ES distributions were estimated. Correlation coefficients also deserve especial attention since they seem not to be as large as they would be expected for stock indices portfolios (tables 3 and 5). This result, also obtained in Mauleón and Perote (1999), is due to the quotation-time-lag among different markets.

Finally, it is also remarkable that one of the main advantages of this approach is the fact that portfolio weights are just one of the parameters of the model and, therefore, the estimates of different portfolios can help risk managers to choose the strategies that minimise VaR at any period depending on weight and risk aversion.

5. Conclusions.

The Edgeworth-Sargan density has been shown capable of fitting financial univariate densities as well as multivariate densities in previous articles (Mauleón and Perote, 1999; 2000). However, unless this distribution were able to capture VaR adequately, its usefulness would be jeopardized, somehow. The purpose of this paper is to show the applicability of this distribution to measure VaR more accurately than through the traditional normal assumption. In this paper, a methodology for computing VaR consistently with the ES distributional hypothesis has been developed and applied to calculate VaR for different portfolios at different confidence levels and in distinct volatility scenarios. For the sake of comparison, VaR under normal

hypothesis has also been computed, hence, the differences between both methodologies represent clear evidence of an understatement of VaR in most cases.

The main facts, supporting the claims of the preceding paragraph, can be summarized as follows:

1.- High frequency financial variables are found to be far from normality, therefore VaR methodology should be modified accordingly to this evidence. Edgeworth-Sargan distribution is able to account for the main empirical features of the underlying density and provides a flexible and simple parametric representation to incorporate into VaR methodology. Moreover, the normal distribution is nested on the Edgeworth-Sargan and thus testing and comparing both distributional hypotheses can be easily analysed.

2.- The variance and covariance matrix of portfolio variables may be estimated consistently with the Edgeworth-Sargan hypothesis. Moreover, portfolio Edgeworth-Sargan distribution can be easily estimated subject to the previous estimation of portfolio variance. Different time varying variance hypotheses, such as a GARCH(1,1), can be also implemented. The computational problems of this complex approach can be solved by accurately selecting the initial values according to the more simple Edgeworth-Sargan structure of the portfolio variables.

3.- VaRs obtained assuming an Edgeworth-Sargan appear to be similar (even smaller) to VaR under normality hypothesis for some portfolios at high confidence levels. Nevertheless, in most cases, the portfolio VaR under ES distribution is clearly bigger. This VaR understatement is especially highlighted for low confidence levels and high volatility scenarios.

4.- This general approach to calculate VaR assuming an ES distribution can help risk managers to accurately measure risk, since the risk measurements can be improved by adding more parameters to the flexible ES parametric formulation. Moreover, optimal weighting policies can be also straightforward developed.

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TABLE 1: Dow Jones index and long run interest rates in United States (low volatility period: 4/1/83 – 28/5/96).

Joint Distributions Of Dow Jones Index And Long Run Interest Rates In United States						
	$w_1 = .1$		$w_1 = .5$		$w_1 = .9$	
	NORMAL	ES	NORMAL	ES	NORMAL	ES
α_{01}	$.7 \times 10^{-6}$	$.68 \times 10^{-6}$	$.7 \times 10^{-6}$	$.66 \times 10^{-6}$	$.7 \times 10^{-6}$	$.68 \times 10^{-6}$
α_{11}	.963	.967	.963	.962	.963	.967
α_{21}	.035	.035	.035	.026	.035	.035
d_{21}		-.055*		-.055*		-.055*
d_{41}		.095		.095		.095
d_{61}		.012		.012		.011
d_{81}		.002		.002		.002
α_{02}	$.12 \times 10^{-5}$	$.66 \times 10^{-6}$	$.12 \times 10^{-5}$	$.68 \times 10^{-6}$	$.12 \times 10^{-5}$	$.66 \times 10^{-6}$
α_{12}	.949	.962	.949	.967	.949	.962
α_{22}	.038	.026	.038	.035	.038	.026
d_{22}		.254		.254		.253
d_{42}		.163		.163		.163
d_{62}		.030		.030		.030
d_{82}		.0026		.0026		.0026
ρ_{12}	-.344	-.297	-.344	-.297	-.344	-.297
Log L	25,722	25,883	25,722	25,883	25,722	25,883
PORTFOLIO						
d_{2p}		.142		.0078*		-.035
d_{4p}		.103		.0537		.066
d_{6p}		.014		.0057		.0074
d_{8p}		.0013		.0009		.0012
Log L		12,321		13,677		12,364
$\sigma_p (.05)$	.00717	.00642	.00441	.00423	.00619	.00643
VaR (.05)	.01183	.01199	.00728	.00704	.0102	.0106
% (.05)		1.30		-3.33		3.99
VaR (.025)	.01399	.01512	.0086	.00876	.0120	.0127
% (.025)		8.13		1.73		5.97
VaR (.01)	.01618	.01942	.01078	.01028	.0144	.0161
% (.01)		16.18		4.83		11.67

* The parameter is not significant at 5% confidence level.

TABLE 2: Footsie index and long run interest rates in United Kingdom (low volatility period: 1/1/90 – 28/5/96).

Joint Distributions Of Footsie Index And Long Run Interest Rates In Uk						
	$w_1 = .1$		$w_1 = .5$		$w_1 = .9$	
	NORMAL	ES	NORMAL	ES	NORMAL	ES
α_{01}	$.37 \times 10^{-6*}$	$.35 \times 10^{-6}$	$.37 \times 10^{-6*}$	$.35 \times 10^{-6}$	$.37 \times 10^{-6*}$	$.35 \times 10^{-6}$
α_{11}	.961	.959	.961	.959	.961	.959
α_{21}	.038	.033	.038	.033	.038	.033
d_{21}		.164		.164		.164
d_{41}		.014*		.014*		.014*
d_{61}		.006*		.006*		.006*
d_{81}		.002*		.002*		.002*
α_{02}	$.38 \times 10^{-6}$	$.48 \times 10^{-6}$	$.38 \times 10^{-6}$	$.48 \times 10^{-6}$	$.38 \times 10^{-6}$	$.48 \times 10^{-6}$
α_{12}	.966	.963	.966	.963	.966	.963
α_{22}	.037	.039	.037	.039	.037	.039
d_{22}		-.037*		-.037*		-.037*
d_{42}		.046		.046		.046
d_{62}		.0006*		.0006*		.0006*
d_{82}		.001		.001		.001
ρ_{12}	-.4515	-.4528	-.4515	-.4528	-.4515	-.4528
Log L	12,906	12,932	12,906	12,932	12,906	12,932
PORTFOLIO						
d_{2p}		-.0166*		.0463*		.101
d_{4p}		.0454		.0429		.0417
d_{6p}		-.0006*		.0068*		.0106
d_{8p}		.0005		.0007*		.0007*
Log L		6,076		6,872		6,255
$\sigma_p (.05)$	.0065	.00647	.00371	.00367	.00624	.00568
VaR (.05)	.01076	.01095	.00642	.00645	.0103	.0099
% (.05)		1.77		.47		-3.05
VaR (.025)	.01272	.0138	.00759	.00786	.0121	.0119
% (.025)		9.13		3.54		-1.59
VaR (.01)	.01519	.01705	.00906	.00964	.0145	.0148
% (.01)		12.21		6.31		1.71

* The parameter is not significant at 5% confidence level.

TABLE 3: Dow Jones and Footsie indices (low volatility period: 1/1/70 – 28/5/96).

Joint Distributions Of Dow Jones And Footsie Indices						
	$w_1 = .1$		$w_1 = .5$		$w_1 = .9$	
	NORMAL	ES	NORMAL	ES	NORMAL	ES

α_{01}	$.14 \times 10^{-5}$	$.58 \times 10^{-6}$	$.14 \times 10^{-5}$	$.58 \times 10^{-6}$	$.14 \times 10^{-5}$	$.58 \times 10^{-6}$
α_{11}	.903	.963	.903	.963	.903	.963
α_{21}	.084	.038	.084	.038	.084	.038
d_{21}		.033*		.033*		.033*
d_{41}		.0905		.0905		.0905
d_{61}		.0142		.0142		.0142
d_{81}		.0018		.0018		.0018
α_{02}	$.71 \times 10^{-6}$	$.13 \times 10^{-5}$	$.71 \times 10^{-6}$	$.13 \times 10^{-5}$	$.71 \times 10^{-6}$	$.13 \times 10^{-5}$
α_{12}	.961	.901	.961	.901	.961	.901
α_{22}	.0395	.086	.0395	.086	.0395	.086
d_{22}		.061*		.061*		.061*
d_{42}		.0412		.0412		.0412
d_{62}		.0064*		.0064*		.0064*
d_{82}		.0004*		.0004*		.0004*
ρ_{12}	.1931	.1802	.1931	.1802	.1931	.1802
Log L	50,029	50,169	50,029	50,169	50,029	50,169
PORTFOLIO						
d_{2p}		.004*		.0066*		.0194*
d_{4p}		.029		.033		.0557
d_{6p}		.0046		.0033		.0086
d_{8p}		.0003		.0004		.00105
Log L		23,487		24,752		23,707
$\sigma_p(.05)$	.00502	.00496	.004857	.00477	.00653	.0064
VaR (.05)	.00829	.008178	.00801	.00787	.0107	.01065
% (.05)		-1.35		-1.74		-1.15
VaR (.025)	.00979	.00974	.00947	.00963	.01274	.01311
% (.025)		-.005		1.68		2.93
VaR (.01)	.01171	.01196	.01132	.01182	.0152	.0162
% (.01)		2.2		4.41		6.67

* The parameter is not significant at 5% confidence level.

TABLE 4: Dow Jones index and long run interest rates in United States (high volatility period: 4/1/83 – 13/11/87).

Joint Distributions Of Dow Jones Index And Long Run Interest Rates In Usa						
	$w_1 = .1$		$w_1 = .5$		$w_1 = .9$	
	NORMAL	ES	NORMAL	ES	NORMAL	ES
α_{01}	$.28 \times 10^{-5}$	$.25 \times 10^{-5}$	$.28 \times 10^{-5}$	$.25 \times 10^{-5}$	$.28 \times 10^{-5}$	$.25 \times 10^{-5}$
α_{11}	.845	.858	.845	.858	.845	.858
α_{21}	.149	.156	.149	.156	.149	.156
d_{41}		.677		.677		.677
d_{81}		.03		.03		.03
α_{02}	$.78 \times 10^{-6*}$	$.47 \times 10^{-7}$	$.78 \times 10^{-6*}$	$.47 \times 10^{-7}$	$.78 \times 10^{-6*}$	$.47 \times 10^{-7}$
α_{12}	.957	.979	.957	.979	.957	.979
α_{22}	.043	.034	.043	.034	.043	.034
d_{42}		.189		.189		.189
d_{82}		.0046		.0046		.0046
ρ_{12}	-.287	-.645	-.287	-.645	-.287	-.645
Log L	8,795	9,576	8,795	9,576	8,795	9,576
PORTFOLIO						
d_{2p}		.168		.318		-.037*
d_{4p}		.107		.108		.0568
d_{6p}		.018		.0075		.0061
d_{8p}		.0017		-.0009*		.0014
Log L		4,233		4,624		4,194
$\sigma_p (.05)$	.00826	.00672	.01205	.01066	.0223	.0228
VaR (.05)	.01363	.01281	.02312	.01989	.0367	.0376
% (.05)		-6.05		16.24		2.43
VaR (.025)	.01611	.01586	.0235	.02873	.0434	.0458
% (.025)		-1.52		22.22		5.53
VaR (.01)	.01924	.02043	.02808	.03429	.0519	.0558
% (.01)		6.13		22.02		7.67

* The parameter is not significant at 5% confidence level.

TABLE 5: Dow Jones and Footsie indices (high volatility period: 1/1/70 – 13/11/87).

Joint Distributions Of Dow Jones And Fotsie Indices						
	$w_1 = .1$		$w_1 = .5$		$w_1 = .9$	
	NORMAL	ES	NORMAL	ES	NORMAL	ES
α_{01}	$.53 \times 10^{-6}$	$.84 \times 10^{-6}$	$.53 \times 10^{-6}$	$.84 \times 10^{-6}$	$.53 \times 10^{-6}$	$.84 \times 10^{-6}$
α_{11}	.968	.959	.968	.959	.968	.959
α_{21}	.037	.046	.037	.046	.037	.046
d_{41}		.385		.385		.385
d_{81}		.0211		.0211		.0211
α_{02}	$.24 \times 10^{-5}$	$.18 \times 10^{-5}$	$.24 \times 10^{-5}$	$.18 \times 10^{-5}$	$.24 \times 10^{-5}$	$.18 \times 10^{-5}$
α_{12}	.886	.876	.886	.876	.886	.876
α_{22}	.095	.112	.095	.112	.095	.112
d_{42}		.0455		.0455		.0455
d_{82}		.0034		.0034		.0034
ρ_{12}	.119	.2586	.119	.2586	.119	.2586
Log L	33,043	33,958	33,043	33,958	33,043	33,958
PORTFOLIO						
d_{2p}		-.014*		-.084		-.086
d_{4p}		.0176		.017		.0079
d_{6p}		.0025		.0002*		-.0048
d_{8p}		.0003		.0003		.00006*
Log L		15,303		16,356		15,712
$\sigma_p (.05)$	.00719	.00737	.00952	.01063	.01504	.01648
VaR (.05)	.01187	.01216	.01571	.01754	.02482	.02719
% (.05)		2.42		11.66		9.57
VaR (.025)	.01403	.01445	.01856	.02083	.02933	.03231
% (.025)		2.95		12.21		1.13
VaR (.01)	.01677	.01724	.02476	.02476	.03505	.03841
% (.01)		2.86		11.64		9.57

* The parameter is not significant at 5% confidence level.

¹ Assuming that the variables have previously been centred and thus have zero mean.

² They do not imply constant covariances, since each variance depends on the time period.

³ For example, assuming that the returns of all assets depend linearly on a benchmark and that the disturbances of the models are uncorrelated simplifies the covariances structure for a portfolio composed of several assets.

⁴ Observe that weights are known at time t .

⁵ Note that constant c_i equals 1 if r_{it} are normally distributed and $1 + 2d_{i2}$, under the Edgeworth-Sargan distribution.

⁶ For the sake of clarity, this notation is used despite the fact that Edgeworth-Sargan distribution does not depend on its first and second moments only.

⁷ In particular, the polynomials used for the empirical examples presented in section 4 are the following: $H_2(v_{it}) = v_{it}^2 - 1$, $H_4(v_{it}) = v_{it}^4 - 6v_{it}^2 + 3$, $H_6(v_{it}) = v_{it}^6 - 15v_{it}^4 + 45v_{it}^2 - 15$ and $H_8(v_{it}) = v_{it}^8 - 28v_{it}^6 + 210v_{it}^4 - 420v_{it}^2 + 105$.

⁸ Notice that the univariate portfolio ES distribution can always be estimated once an estimation of the variance and covariance matrix has been obtained. Moreover, the dimensionality of the portfolio can also be simplified, since individual assets can be grouped into indices.

⁹ This is the reason why zero mean variables are assumed. Also note that the AR(1) structure imposed to the conditional mean of each variable could have been estimated jointly with the rest of the parameters of the model, nevertheless, they were estimated separately to facilitate the convergence of the optimisation algorithms.

¹⁰ As the multivariate normal is nested on the multivariate ES, a simple LR test can be implemented straightforward to reject the normality assumption.

¹¹ In this sense, it is remarkable the evidence found by Perote (1999) and Mauleón and Perote (2000) showing that the ES distribution is able to capture the behaviour of high frequency financial variables even better than other simpler parametrical structures, such as the Student's t.

¹² Notice that the use of a general ES structure can also be interpreted as a semi-non-parametric approach to the underlying density.