Cash, Paper-based and Electronic Payments: A Theoretical Approach*

Francisco Callado‡, Jana Hromcová† and Natalia Utrero

Abstract

In this paper we provide a general equilibrium model that helps explaining payment choice at the retail level: cash, electronic and paper-based instruments. In particular, it provides theoretical foundations to reconcile previous empirical evidence on the matter. Payment pattern of a given country can be shaped by the payment infrastructure, the cost of each payment instrument, the degree of technology development and the interest rate. We show that the introduction of a cheaper payment instrument, in this case electronic payments, may be welfare improving. The calibration exercise for Norway illustrates that the policy of correct pricing of checks promoted by the Norwegian authorities may imply 4% increase in the welfare of the country.

Keywords: cash; payments; human capital; cash-in-advance;
JEL classification: E42, E41, O42

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‡Universitat de Girona, Departament d’Economia i Empresa, Campus de Montilivi, 17071 Girona, Spain; Tel: +34 97 241 9883; Fax: +34 97 241 8032; franciscojose.callado@udg.edu.

†Corresponding author. Universitat Autònoma de Barcelona, Departament d’Economia Aplicada, Edifici B, 08193 Bellaterra, Barcelona, Spain; Tel: +34-93 581 4579; Fax: +34-93 581 22 92; jana.hromcova@uab.cat
1 Introduction

There is a large empirical literature on payment systems, yet few theoretical papers analyze payment instrument choice. This paper develops a model to analyze the dynamics of the payment instruments choice, in particular the behavior of cash, paper-based and electronic payments. The variables that can explain differences between countries are the payment infrastructure, relative cost of the available payment instruments, degree of technology development and monetary policy.

The way in which payments are made in a given country happens to be a relevant issue at least in two respects. On one side, the cost of the complete payment system, that could account up to 3% of GDP, Humphrey, Pulley and Vesala (1996 and 2000), is clearly affected by agents’ choice since the cost of each payment instrument differs. On the other hand, payment choice could influence the functioning of the financial system and therefore facilitate trade in the real economy. In fact these decisions could have important economic consequences since a relevant part of the GDP (around 2/3 in the US) comes from consumer transactions and these are completed with some method of payment, Schreft (2006).

The continuous evolution of information technology (IT) has led to a significant transformation of payments industry, Evans and Schmalensse (2009). However, the development of new payment instruments does not imply the elimination of traditional paper-based methods. Consumers ultimately determine which of these instruments they actually use. Moreover, once consumers and merchants get comfortable with a particular technology, they need a compelling reason to switch to another. First of all, for a new method of payment to be successful, it must attract substantial number of users, offer significant cost savings or added convenience relative to existing payments technologies or methods. Second, whatever its cost or convenience, a payment system must be trustworthy and secure, or people will not use it, Litan and Baily (2009). National regulations can provide comfort to users and thereby, accelerate the acceptance of a particular payment instrument. Therefore, there is a case for active public policy with regard to payment system design and security, Stojanovic (2001).

The distribution and evolution of payment instruments use is usually very different across countries, even within developed countries, see Callado and Utrero (2004). There is a growing body of empirical literature focusing on the characteristics of this distribution and analyzing the possible determinants of these differences. Starting with Humphrey et al. (1996) who examine the payment systems of 14 developed countries and find that the use of electronic means of payment is clearly increasing in all countries. Besides, they try to explain the possible factors behind the different structures across countries. Among these they find as the most important the degree of payment availability (number of users, terminals, etc.) and institutional and cultural differences (income, new payment instruments, etc.). Similarly, Callado and Utrero (2007) analyze the case of European emerging economies and find that the use of cash, although reducing, is still persistent. Hancock and Humphrey (1998) provide evidence of how electronic means of payment (credit and debit cards) gain importance with respect to checks and cash in many developed countries between 1987 and 1993. In Figure 1 we show similar behavior in the data in more recent period, between 1996 and 2001.

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1Humphrey et al. (1996) claim that the cost of electronic payments ranges from 1/2 to 1/3 to that of paper-based ones. Estimates set this proportion equal to 28.77% in Norway (bank cost) for 1994 and 44.71% (total cost) in the US for 1993, Humphrey, Kim and Vale (2001).

2Stojanovic (2001) argues that the adoption of new payment instruments, and e-money in particular, contributes to cash substitution and the development of more efficient payment and banking systems.

3Data on payment systems comes from Norges Bank, Bank of International Settlements and European Central Bank.
Figure 1: Transformation of the payment methods in some European countries and the US between 1996 and 2001.

An interesting finding is that the payment choice behavior of consumers in the US is a bit different than that of the rest of developed countries. Although cash use is small, the relevance of electronic payments is clearly lower. Humphrey (2004) deepens the analysis of the US by analyzing the substitution of cash by cards. Data presented in his study again show how the relative importance of electronic means of payments (credit and debit cards) increases over time.

Humphrey et al. (2001) include price in trying to explain different consumer choices observed. Using data from Norway, they find empirical evidence that technology development and relative prices, together with the relationship between cost and technology, could have a word to say in explaining these differences. Accordingly, although not empirically analyzed, differences in technology adoption and diffusion may affect consumer choices as well.

Despite the relevance of payment systems and the empirical interest on the matter, there is a lack of theoretical literature concerning wider choice of payment instruments, see Schreft (2006) for example. Many theoretical models which consider two payment instruments were developed. For example, Schreft (1992), Gillman (1993), Aiagari, Braun and Eckstain (1998) show that the choice between cash and a payment via intermediary depends on the mix of the cost of the alternative mean of payment and the monetary policy. Ireland (1994a) points out that the decrease in the use of cash and its substitution by cards is caused by an increase in income. Markose and Loke (2003) show that this substitution is also due to the availability of payment terminals at the point of sale. Ireland (1994b), Marquis and Refett (1994), English (1999) and Hromcová (2008) relate the choice of payment instruments also to the technological progress.

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4 High usage of paper-based payments in the US may be rooted in the fact that checks were generally accepted and used as a default payment method since the 16th century, see the historical survey in Quinn and Roberds (2008).

5 Many theoretical models which consider two payment instruments were developed. For example, Schreft (1992), Gillman (1993), Aiagari, Braun and Eckstain (1998) show that the choice between cash and a payment via intermediary depends on the mix of the cost of the alternative mean of payment and the monetary policy. Ireland (1994a) points out that the decrease in the use of cash and its substitution by cards is caused by an increase in income. Markose and Loke (2003) show that this substitution is also due to the availability of payment terminals at the point of sale. Ireland (1994b), Marquis and Refett (1994), English (1999) and Hromcová (2008) relate the choice of payment instruments also to the technological progress.
they use. We let our consumers use cash, paper-based or electronic payments. The main point is that a consumer facing a decision on the mean of payment will take into account the relative cost of each instrument. We adapt for our purpose the model of Hromcová (2008). In this model, technology improvement happens via accumulation of human capital through studying and the cost of each payment instrument depends on the place and time of the transaction. We consider that a necessary infrastructure for electronic payments is available before it is actually used. However, electronic transactions do not emerge until the economy is ready to use them (apart from being trustworthy they must be cheap enough). We study how a monetary policy or an authority’s action that favors some of the available means of payments alters agents payment instrument choice and welfare. The model is calibrated using data from the US and Norway economies between 1991 and 2007. We compare the dynamics of the payment choice in the model and the data for both countries. The model can generate very similar behavior to the one found in the data and explain changes in the payment behavior due to variations in the payment infrastructure, relative cost of payment instruments, degree of technology development and monetary policy.

We evaluate welfare effects of changes in various factors that influence the choice of the payment methods. We find that a policy which promotes the usage of electronic instruments may lead to important welfare gains. For the case of Norway we obtain that by pursuing a policy of direct pricing of checks according to their costs about 4% increase in the stream of consumption can be achieved. A recommendation implied by our results is to price payment instruments based on their costs, so that cheap instruments are chosen and resources are allocated more efficiently.

The remainder of the paper is organized as follows. In section 2 we use data of payment patterns of US and Norway to illustrate differences in the recent evolution of the payment instrument choice. The model and its main properties are stated in section 3. In section 4 we describe the balanced growth path behavior. In section 5 we discuss the behavior in the transition and the reaction of the economy to changes in different parameters. Welfare effects of earlier or later arrival of the electronic era are studied in section 6. Final conclusions are summarized in section 7.

2 Payment Patterns in the US and Norway

The main point of our model is that the relative cost of alternative means of payment is of relevance to make payments decisions, and therefore could affect a country’s economic outcome. Pricing payment services along their true cost would make consumers choose the most efficient payment instrument. However, traditionally, consumers do not face directly payment costs, making their decisions on lost interest on transaction balances. Interestingly, some countries have started to directly price payment instruments according to their costs. This is the case of Norway. This pricing policy makes very appealing the use of Norwegian data for our analysis. Figure 2 shows the evolution of prices of alternative payment instruments in Norway between 1991 and 2007.
Following the changes in Norwegian pricing policy in 1995, checks have increased their relative cost from three times to nearly five times more than the second more expensive instrument, cash. Accordingly, this pricing policy have impacted consumers’ behavior very rapidly, consumers have reduced their use of checks. Norway switched radically from checks and cash towards electronic payments, mostly after 1995. The growth in the relative usage of electronic payments is very high. In 17 years Norway transformed from an economy that practically did not use electronic payments at all to a one where electronic transactions represent 75% of all payments, see Figure 3.

The case of the US is also interesting but for different reasons. Data on prices are lacking but as stated above, the distribution of payment instruments is slightly different from other developed countries. The relative usage of checks employed in the US presents a slightly negative trend although remains quite stable until 2001. Later on the decrease is faster. Despite of this decrease, the use of checks in the US is still relevant for the US economy since the US checking system is – even with the recent Check 21 Act modification – the oldest, the slowest, the most expensive and easily the most complex of the payment devices in use, Felsenfeld and Bilali (2005). The usage of electronic payments exhibits a steady increase except between 2001-2003. Cash share decreases steadily until 2001 when a slight increase in its relative usage is observed. An interesting insight of the comparison between Norway and the US is that Norwegian usage of electronic payment instruments quickly surpasses that of the US, even if in 1991 it lagged far behind. As for checks, after 17 years Norway practically does not use any paper-based payments while in the US checks remain one of the most important means of payments. One possible explanation is the direct pricing of checks in Norway and the neglecting of pricing according to their true cost in the US. Therefore, the introduction of costs and prices is crucial for the analysis and can explain recent payment trends.

For instance, in 1993 an electronic payment (ACH) had a cost $1.31, which is only 45% of the $2.93 total expense for an analogous paper-based payment (check) in the US, Wells (1996). For Norway, in 1994 the average bank cost of a check payment was $2.15, 3.4 times greater than the $0.69 for cards, Humphrey et. al. (2001).
Figure 3: Evolution of the share of cash, checks and cards (credit and debit) in consumer payments in the USA and Norway for the interval 1991-2007.

3 Model

3.1 The household problem

We follow closely the specification of the economy in Hromcová (2008). The economy consists of a large number of infinitely lived households. All households have identical preferences, production and trade opportunities.

Households inhabit the following environment: they face continuum of spatially separated markets, which are indexed by $i \in [0, 1]$. All households live in market $0$, and the index $i$ indicates the distance from home. In each market $i$ a distinct perishable good is produced and sold in every period. Goods are thus indexed by $i$, which corresponds to the market of both production and trade. The economy has a representative household with preferences given by the utility function

$$U_t = \beta^t \int_0^1 c_t(i)^{1-\theta} \left(\frac{1}{1-\theta}\right) di$$

(1)

where $c_t(i)$ is defined as the consumption at period $t$ of the good produced in market $i$ and $\theta > 0$ is the inverse of the elasticity of intertemporal substitution. The production and trade is like in Lucas and Stokey (1983). Each household is composed of a worker-shopper pair.

Prior to any trading, government fixes the gross nominal interest rate $R$ to be constant in all periods. We will assume that $R \geq 1$. Agents enter the period $t$ with certain amount of monetary balances $M_t$ and debt $B_t$, carried over from the previous period, and human capital stock $h_t$. A representative worker decides to produce on any of the markets $i$ via the net production function

$$y_t = wh_t l_t$$

(2)
where $w$ denotes the marginal product of human capital in goods production, and $l_t$ is the amount of time spent working. Human capital accumulation depends on the time spent studying, on the level of human capital and on the depreciation rate according to

$$h_{t+1} = \phi (1 - l_t) h_t + (1 - \delta) h_t$$

(3)

where $\phi$ is the efficiency of learning parameter and $\delta$ is the depreciation rate.

First, the goods market opens and consumption takes place. Worker stays at the market $i$ during the whole period. Shopper visits various markets to acquire consumption goods carrying all the monetary balances of the household.

To acquire consumption goods agents can use government issued money or alternative means of payment. When using non-cash payments, they can choose from paper-based or electronic instruments. All goods purchased with government issued money will be referred to as cash goods. Goods purchased via paper-based payments will be referred to as paper-based goods and goods purchased via electronic payments will be referred to as electronic goods. Goods only differ in the way they are acquired.

Nominal monetary balances $M_t$ can be used to buy goods in some of the markets indexed by $i$. Cash purchases are subject to the liquidity constraint

$$\int_0^1 [1 - \xi_t(i)] c_t(i) di \leq \frac{M_t}{p_t},$$

(4)

where $p_t$ is the level of prices and $\xi_t(i) = 0$ if a good is purchased on market $i$ with cash, or $\xi_t(i) = 1$ if a good is purchased on market $i$ via a non-cash payment. The financial intermediary enables paper-based and electronic payments at a cost that is given for each market $i$ and period $t$.

After the consumption takes place, the monetary holdings of agents are augmented by a lump sum transfer $X_t$ from the government. The amount $X_t$ is endogenously determined in the system according to the given nominal interest rate, so that the money demand is totally satisfied. As the next step the securities market opens. During the securities trading session households choose their currency holdings $M_{t+1}$. They also purchase (or issue) one-period nominally denominated pure discount bonds paying $B_{t+1}$ units of money at period $t + 1$ while they cost $B_{t+1}/R$ units of money at period $t$. Bonds are in zero net supply. The budget constraint agents are facing can be written

$$\int_0^1 c_t(i) di + \int_0^1 \xi_t(i) \left\{ \nu_t(i) \gamma_t^\sigma(i) + [1 - \nu_t(i)] \gamma_t^e(i) \right\} di + \frac{M_{t+1}}{p_t} + \frac{B_{t+1}}{Rp_t} \leq wh_t + M_t + B_t + \frac{X_t}{p_t},$$

(5)

where $\xi_t(i) = 1$ for a non-cash purchase at market $i$, $\nu_t(i) = 0$ if the good is purchased on market $i$ via an electronic payment and $\nu_t(i) = 1$ if the good is bought using a paper-based private security. The real cost of the non-cash transaction at time $t$ on market $i$ is $\gamma_t^\sigma(i)$ and $\gamma_t^e(i)$ for paper-based and electronic payments, respectively.
3.2 Cost of alternative means of payments

As stated above, the relative cost of alternative means of payment could be of relevance to a country’s economic outcome. Pricing payment services according to their cost would make consumers choose the lowest cost payment instrument. This has not been the case, consumers traditionally pay their payment services through lost interest on transaction balances although there are few exceptions with appealing results like Norway. In this model we will follow this option and assume that the intermediation cost must be paid by the buyer, Ireland (1994b). This allows us to evaluate different effects of alternative pricing policies found across countries and to link the characteristics of the economy and the financial system to consumer payments’ decision making.

The existence of this intermediation cost comes from the fact that, to be able to purchase without cash, some resources must be devoted to making the non-cash payment itself available and reliable, to checking the identity of the buyer and his ability to pay. This cost is going to be proportional to the distance to home market and to the value of purchase. When the shopper is far away from home (market zero) the communication becomes more difficult, and therefore the payment to the intermediary increases with $i$.\footnote{In similar terms, the US Monetary Control Act of 1980 explains the need to develop a nationwide payment services for all depository institutions on grounds of size and location: services should include smaller institutions in remote locations, where volumes are typically low and costs are high, Chakravorti et al. (2005).}

Besides, higher purchase means that more importance will be given to checking the ability of the buyer to pay and again the cost would be greater. In fact, banking infrastructure and value of transaction have been identified as relevant variables for the choice of the type of instrument, Humphrey et al. (1996), Humphrey et al. (2001), Callado and Utrero (2004). Technology and economic development are also important issues in this choice.

The real payment made to the intermediary is characterized by a function that fulfills these and other properties found in some empirical studies, see Hromcová (2008): the intermediation cost is lower in richer countries, the cost of intermediated payment diminishes over time, and the cost elasticity is close to zero (which also motivates the proportional intermediation cost). The process of human capital accumulation gives a potential for the development of new technologies. This development will decrease the processing cost of both paper-based and electronic payments.

We specify the intermediation costs for electronic and paper-based payments in the following way

$$\gamma_{t}^{e}(i) = \left[ \gamma(i) + F \right] \frac{1}{(h_{t})^{\alpha} t_{i}} c_{t}(i), \quad (6)$$

$$\gamma_{t}^{p}(i) = \gamma(i) \frac{1}{(h_{t})^{\alpha} c_{t}(i)}. \quad (7)$$

Both paper-based and electronic payments have a time independent cost $\gamma(i)$ that is increasing with the distance from home, similarly as in Gillman (1993), Ireland (1994a) or Hromcová (2003), for example. That is, payment availability and reliability will decrease with distance and this is similar both in paper and electronic payments. For the sake of tractability we assign it the functional form used previously in the literature, introduced in Ireland (1994a)

$$\gamma(i) = \frac{i}{1 - i}. \quad (8)$$

The fixed cost $F$ is market and time independent and it is attributed to the necessary investment for a system to be capable of operating the electronic transactions. We can think of different aspects
that are intuitively related to this $F$, such as high enough number of stores interested in electronic payments, consumer’s readiness to use them or acquiring and maintaining of a machine that enables electronic transactions. This cost can be interpreted as a barrier to the electronic era.\(^8\)

The time dependent part of the intermediation cost,

$$\frac{1}{(h_t)^{\alpha_j}} c_t^j(i), \ j = \varphi \ or \ e,$$

embodies properties concerning the proportionality to consumption purchases and the effect of new technologies on the cost. It decreases as the level of knowledge increases and it is proportional to consumption purchases done via intermediary. The function $\frac{1}{h_t}$ reflects the following: as more human capital is accumulated, more knowledge is available, better technologies can be developed and cheaper intermediation services can be offered. The parameter $\alpha_j$ can be interpreted as the degree of knowledge diffusion into the payment system.\(^9\) We assume that $\alpha_j > 0$. Similarly to Acemoglu and Zilibotti (2001) we can rank both technologies by capital intensity, where in the definition of capital we include both human and technology diffusion. Therefore, using certain technology efficiently entails an appropriate human capital development and an appropriate degree of knowledge diffusion. In this vein, paper based payment instruments are less capital intensive than electronic ones. Then, the degree of knowledge diffusion into the electronic payments is higher than the knowledge diffusion into the paper-based transactions, $\alpha_e > \alpha_\varphi$. As the economy grows and new technologies are developed, both payments get cheaper, but the electronic payments, more capital intensive, get cheaper in a faster pace.

The mean of payment will be chosen according to its cost. The opportunity cost of buying with cash is the nominal interest rate $R - 1$, which does not vary across markets. The opportunity cost of buying with paper-based or electronic securities is the one corresponding to the intermediation cost $\gamma^\varphi_t(i)$ and $\gamma^e_t(i)$, respectively. Figure 4 shows the behavior of the opportunity cost of using cash, paper-based and electronic payments across markets. For an economy at a low stage of development (low level of human capital) it may happen that no electronic goods will be employed, because they are too expensive. In Figure 4a we can observe that in all markets with indexes below $\chi_t$ paper-based payments have the lowest cost. In all market with indexes above $\chi_t$ cash will be employed because the opportunity cost of holding money is lower than the intermediation cost for either non-cash alternative. For a more developed economy, Figure 4b, paper-based payments are the cheapest way of acquiring consumption goods in all markets with indexes below $\chi_t$. In all markets with indexes between $\chi_t$ and $s_t$ electronic payments have the lowest cost, and in markets with indexes above $s_t$ government issued money are the cheapest.

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\(^8\)In particular $F$ can be shaped by at least two kinds of dimensions. First, the minimum necessary infrastructure for a payment system to operate and second the geographical factors, in the spirit of Galor and Weil (2000), that condition the development and implementation of this minimum infrastructure necessary to make electronic transactions.

\(^9\)For $h_t < 1$ higher $\alpha_j$ increases the intermediation cost for a given level of human capital. We can understand that in our analysis countries capable of performing electronic transactions are developed enough so the initial level of human capital $h_t > 1$. 

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Figure 4: Determination of the marginal markets: when \( i = \bar{\chi}_t \), agents are indifferent between buying with cash or paper-based securities; when \( i = \chi_t \), agents are indifferent between buying via electronic or paper-based transactions; when \( i = s_t \), agents are indifferent between buying with cash or via electronic payments. (a) Low development: only cash and paper-based payments are used; (b) Middle development: all three payment instruments are used.

We arbitrarily assume that cash will be used on the marginal markets \( \bar{\chi}_t \) and \( s_t \), and electronic payment on the cutoff market \( \chi_t \). There will exist a moment in time for which all three marginal markets coincide \( s_t = \bar{\chi}_t = \chi_t \). Thus by checking the relationship between \( s_t \) and \( \chi_t \) we can recognize the mentioned two stages of development. One in which no electronic goods will be employed, \( s_t < \chi_t \), and the stage of the coexistence of all three types of considered means of payments. The basic characteristic of the second stage is that in the course of time the electronic private securities will drive out of the markets cash and paper-based payments.

Define

\[
c_t(i) = \begin{cases} 
    c^0_t(i) & \text{when } \xi_t(i) = 0, \\
    c^e_t(i) & \text{when } \xi_t(i) = 1 \text{ and } \nu_t(i) = 0, \\
    c^e_t(i) & \text{when } \xi_t(i) = 1 \text{ and } \nu_t(i) = 1.
\end{cases}
\]

The functions \( c^0_t(i) \), \( c^e_t(i) \) and \( c^e_t(i) \) characterize the cash, paper-based and electronic consumption per market \( i \), respectively. Recognizing the two stages of development, we can rewrite the utility function

\[
\sum_{t=1}^{\infty} \beta^t \left[ \int_0^{\min\{\bar{\chi}_t, s_t\}} \frac{c^0_t(i)^{1-\theta} - 1}{1-\theta} di + \int_{\min\{\bar{\chi}_t, s_t\}}^{\max\{\bar{\chi}_t, s_t\}} \frac{c^e_t(i)^{1-\theta} - 1}{1-\theta} di + \int_{\max\{\bar{\chi}_t, s_t\}}^{1} \frac{c^e_t(i)^{1-\theta} - 1}{1-\theta} di \right],
\]

(9)
the budget constraint

\[
\min_{\tilde{x}_t, x_t} \int_0^1 [c_t^e(i) + \gamma_t^e(i)] \, di + \max_{\tilde{x}_t, s_t} \int_{\tilde{x}_t, x_t} [c_t^e(i) + \gamma_t^e(i)] \, di + \int_{\tilde{x}_t, x_t} c^0_t(i) \, di + \frac{M_{t+1}}{p_t} + \frac{B_{t+1}}{R_p} \leq w h_t l_t + \frac{M_t}{p_t} + \frac{B_t}{p_t} + X_t = 1 \tag{10}
\]

and the cash-in-advance constraint

\[
\int_{\tilde{x}_t, x_t} c^0_t(i) \, di \leq \frac{M_t}{p_t}. \tag{11}
\]

### 3.3 Equilibrium

**Definition:** Given the set of initial conditions \(h_1, M_1, B_1\) and the nominal interest rate \(R\), the equilibrium consists of sequences \(\{c_t^0(i), c_t^e(i), c_t^e(i), l_t, h_{t+1}, M_{t+1}, B_{t+1}, \chi_t, \tilde{x}_t, s_t, X_t, p_t\}^{\infty}_{t=1}\) such that

(a) a representative household is maximizing the discounted utility (9) subject to the budget constraint (10), the cash-in-advance constraint (11) and the condition for accumulation of human capital (3), choosing the sequences \(\{c_t^0(i), c_t^e(i), c_t^e(i), l_t, h_{t+1}, M_{t+1}, B_{t+1}, \chi_t, \tilde{x}_t, s_t\}^{\infty}_{t=1}\);

(b) markets for goods, money and bonds clear in every period,

\[
\min_{\tilde{x}_t, x_t} \int_0^1 c_t^e(i) \, di + \max_{\tilde{x}_t, x_t} \int_{\tilde{x}_t, x_t} c_t^e(i) \, di + \int_{\tilde{x}_t, x_t} c^0_t(i) \, di + \int_{\tilde{x}_t, x_t} \gamma_t^e(i) \, di + \int_{\tilde{x}_t, x_t} \gamma_t^e(i) \, di = w h_t l_t, \tag{12}
\]

\[
M_{t+1} = M_t + X_t, \tag{13}
\]

\[
B_{t+1} = 0. \tag{14}
\]

Let \(\lambda_t, \eta_t, \text{ and } \tau_t\) be the non-negative Lagrange multipliers associated with the budget constraint (10), the cash-in-advance constraint (11), and the condition for accumulation of human capital (3), respectively. The equations that characterize the equilibrium are the above mentioned market clearing conditions (12), (13), (14) and the first order conditions on all types of consumptions, labor,
capital, nominal balances, nominal bonds and marginal markets indexes, respectively,

\[ c^0_t(i)^{-\theta} = \lambda_t + \eta_t, \]  
\[ c^e_t(i)^{-\theta} = \lambda_t, \]  
\[ c^e_t(i)^{-\theta} = \lambda_t, \]  
\[ \lambda_t w = \tau_t \phi, \]  
\[ c^0_t(\bar{x}_t) \frac{1^{1-\theta} - 1}{1-\theta} - c^e_t(\bar{x}_t) \frac{1^{1-\theta} - 1}{1-\theta} = -\lambda_t [c^e_t(\bar{x}_t) + \gamma_t(\bar{x}_t)] + (\lambda_t + \eta_t) c^0_t(\bar{x}_t), \]  
\[ c^e_t(s_t) \frac{1^{1-\theta} - 1}{1-\theta} - c^0_t(s_t) \frac{1^{1-\theta} - 1}{1-\theta} = -\lambda_t [c^e_t(s_t) + \gamma_t(s_t)] + \lambda_t [c^e_t(s_t) + \gamma_t(s_t)] \]  
\[ c^e_t(\chi_t) \frac{1^{1-\theta} - 1}{1-\theta} - c^0_t(\chi_t) \frac{1^{1-\theta} - 1}{1-\theta} = -\lambda_t [c^e_t(\chi_t) + \gamma_t(\chi_t)] + \lambda_t [c^e_t(\chi_t) + \gamma_t(\chi_t)] \]  

Using (15), (20) and (21), we can rewrite the first order condition on cash consumption as

\[ c^0_t(i)^{-\theta} = R\lambda_t. \]  

From the first order conditions on the marginal markets between paper-based and cash goods (22), electronic and cash goods (23) and between paper-based and electronic goods (24) we get 10

\[ \gamma^e_t(\bar{x}_t) = \frac{1}{\lambda_t} \left[ \frac{c^e_t(\lambda_t) \frac{1^{1-\theta} - 1}{1-\theta} - c^0_t(R, \lambda_t) \frac{1^{1-\theta} - 1}{1-\theta}}{1} \right] + Rc^0_t(R, \lambda_t) - c^e_t(\lambda_t), \]  
\[ \gamma^e_t(s_t) = \frac{1}{\lambda_t} \left[ \frac{c^e_t(\lambda_t) \frac{1^{1-\theta} - 1}{1-\theta} - c^0_t(R, \lambda_t) \frac{1^{1-\theta} - 1}{1-\theta}}{1} \right] + Rc^0_t(R, \lambda_t) - c^e_t(\lambda_t), \]  
\[ \gamma^e_t(\chi_t) = \gamma^e_t(\chi_t). \]  

Using the given forms of the cost functions (6) and (7), we get for the respective cutoff markets

\[ \bar{x}_t = \bar{x}(R, h_t) = \frac{\Theta(R)}{\left(1 + \Theta(R) / (h_t)^{\alpha_\phi}\right)}, \]  
\[ s_t = s(R, h_t) = \frac{\Theta(R) - \frac{1}{(h_t)^{\alpha_e}} F}{\left(1 + \Theta(R) - \frac{1}{(h_t)^{\alpha_e}} F\right)}, \]  
\[ \chi_t = \chi(h_t) = \frac{F}{(h_t)^{\alpha_e - \alpha_\phi} + F - 1}. \]  

10Note that the initial level of the Lagrange multiplier on the budget constraint depends on the monetary policy, so in fact $\lambda_t = \lambda_t(R)$. We write just $\lambda_t$ in order to keep the notation as simple as possible.
where
\[
\Theta (R) = \begin{cases} 
\ln R & \text{for } \theta = 1, \\
\frac{\theta}{1 - \theta} \left( 1 - \frac{1}{R^{1-\theta}} \right) & \text{for } \theta \neq 1.
\end{cases}
\] (32)

As can be seen from the goods market equilibrium (12), the current period output is spent between paper-based credit consumption, electronic credit consumption, cash consumption and payments to the intermediary for both types of non-cash purchases. The real monetary balances, which equal the amount of cash consumption purchased in all markets, are
\[
m_t = m(R, h_t, \lambda_t) = \begin{cases} 
[1 - \bar{\chi}(R, h_t)] c^0(R, \lambda_t) & \text{for } s_t \leq \chi_t, \text{ or} \\
[1 - s(R, h_t)] c^0(R, \lambda_t) & \text{for } s_t > \chi_t
\end{cases}
\] (33)

where
\[
m_t = \frac{M_t}{p_t}.
\] (34)

The paper-based consumption in all markets is
\[
\varphi_t = \varphi(R, h_t, \lambda_t) = \begin{cases} 
\bar{\chi}(h_t) c^\varphi(\lambda_t) & \text{for } s_t \leq \chi_t, \text{ or} \\
\chi(R, h_t) c^\varphi(\lambda_t) & \text{for } s_t > \chi_t
\end{cases}
\] (35)

and electronic consumption in all markets
\[
e_t = e(R, h_t, \lambda_t) = \begin{cases} 
0 & \text{for } s_t \leq \chi_t, \text{ or} \\
[s(R, h_t) - \chi(R, h_t)] c^e(\lambda_t) & \text{for } s_t > \chi_t.
\end{cases}
\] (36)

The part of output that goes to the intermediary is the sum of payments corresponding to paper-based consumption (solving for the fourth integral in the goods market equilibrium (12)),
\[
\Gamma_t^p = \Gamma^p(R, h_t, \lambda_t) = \begin{cases} 
\{- \bar{\chi}(h_t) - \ln [1 - \bar{\chi}(h_t)]\} c^\varphi(\lambda_t) & \text{for } s_t \leq \chi_t, \text{ or} \\
\{- \chi(R, h_t) - \ln [1 - \chi(R, h_t)]\} c^\varphi(\lambda_t) & \text{for } s_t > \chi_t.
\end{cases}
\] (37)

and the one corresponding to electronic consumption
\[
\Gamma_t^e = \Gamma^e(R, h_t, \lambda_t) = \begin{cases} 
0 & \text{for } s_t \leq \chi_t, \text{ or} \\
\{[\chi_t - s_t + \ln (1 - \chi_t) - \ln (1 - s_t)] + [s_t - \chi_t] F \} \frac{1}{(h_t)^{\alpha_2}} c^e(\lambda_t) & \text{for } s_t > \chi_t.
\end{cases}
\] (38)

The monetary transfer from the government is given by
\[
X_t = (\mu_{t+1} - 1) M_t
\]
where \( \mu_{t+1} \) is the gross growth rate of money supply between period \( t \) and \( t + 1 \). From (18) and (19) we get the evolution of the marginal utility of consumption
\[
\frac{\lambda_t}{\lambda_{t+1}} = \beta (\phi + 1 - \delta).
\] (39)
Combining expressions (21), (34) and (39) we can write for the growth rate of money supply

\[ \mu_{t+1} = \frac{R}{(\phi + 1 - \delta)} \frac{m_{t+1}}{m_t}. \]  

(40)

Notice that the constant in (40) is the gross inflation rate

\[ \pi_{t+1} = \frac{p_{t+1}}{p_t} = \frac{R}{(\phi + 1 - \delta)}. \]  

(41)

4 Balanced Growth Path

To perform our analysis we will assume that human capital grows at a positive rate, i.e.

\[ \frac{h_{t+1}}{h_t} > 1. \]

To characterize the properties of the economy when it reaches the balanced growth path we look at the behavior of variables as time goes to infinity. From (29)-(31) we see that when the human capital accumulates at a positive rate and \( h_t \to \infty \) the cutoff index between the electronic and cash goods and the one between paper-based and cash approach unity, \( s(R, h_t) \to 1 \) and \( \chi(R, h_t) \to 1 \) respectively, while the cutoff index between the paper-based and electronic payments is approaching zero, \( \chi(h_t) \to 0 \). That means that the electronic payments are used in more and more markets and the cash and paper-based private securities are less and less employed.

Let us look at the long run growth rates of all types of consumption and payments to the intermediary. When taking the limits of the growth rates of these variables as time goes to infinity we get the following: cash consumption and the payment to the intermediary for electronic transactions in the long run grow at the rate

\[ \lim_{t \to \infty} \left( \frac{m_{t+1}}{m_t} \right) = \lim_{t \to \infty} \left( \frac{\Gamma_{t+1}^e}{\Gamma_t^e} \right) = \left( \frac{\lambda_t}{\lambda_{t+1}} \right)^{1/\bar{\theta}} \left( \frac{h_t}{h_{t+1}} \right)^{\alpha_e}, \]  

(42)

paper-based consumption

\[ \lim_{t \to \infty} \left( \frac{\varphi_{t+1}}{\varphi_t} \right) = \left( \frac{\lambda_t}{\lambda_{t+1}} \right)^{1/\bar{\theta}} \left( \frac{h_t}{h_{t+1}} \right)^{\alpha_e - \alpha_{\varphi}}, \]

the intermediation cost for paper-based payments in the long run does not grow

\[ \lim_{t \to \infty} \left( \frac{\Gamma_{t+1}^\varphi}{\Gamma_t^\varphi} \right) = 1 \]  

(43)

and the electronic consumption grows at the rate

\[ \lim_{t \to \infty} \left( \frac{e_{t+1}}{e_t} \right) = \left( \frac{\lambda_t}{\lambda_{t+1}} \right)^{1/\bar{\theta}}. \]
Goods market equilibrium

\[ \frac{w_t}{h_t} = \frac{m_t}{h_t} + \frac{\varphi_t}{h_t} + \frac{e_t}{h_t} + \frac{\Gamma^\varphi_t}{h_t} + \frac{\Gamma^e_t}{h_t} \]  

(44)

implies that electronic consumption in the long run must grow like human capital. That means that

\[ \frac{h_{t+1}}{h_t} = \left( \frac{\lambda_t}{\lambda_{t+1}} \right)^{\frac{1}{\bar{\sigma}}} \]  

(45)

Plugging (45) into (42) and using (39), (3) and (40) we can summarize the results in the following proposition.

**Proposition 1** On the balanced growth path cash consumption and the payment for electronic transactions grow at the rate

\[ \left[ \beta (\phi + 1 - \delta) \right]^{1 - \alpha_e} \]

paper-based consumption grows at

\[ \left[ \beta (\phi + 1 - \delta) \right]^{1 - \alpha_e + \alpha_\varphi} \]

electronic-based consumption and human capital grow at the rate

\[ \left[ \beta (\phi + 1 - \delta) \right]^{1 / \bar{\sigma}} \]

labor is given by

\[ \frac{(\phi + 1 - \delta) - \left[ \beta (\phi + 1 - \delta) \right]^{1 / \bar{\sigma}}}{\phi} \]

and the growth rate of money supply is

\[ R \beta^{1 - \alpha_e - \theta} (\phi + 1 - \delta) \left[ \beta (\phi + 1 - \delta) \right]^{1 - \alpha_e - \theta} \]

**Proof.** The labor on the balanced growth path is obtained by equating the condition for the human capital accumulation (3) and the long run growth rate of capital. ■

5 **Transitional Dynamics**

5.1 **Numerical technique**

In order to characterize the dynamics we rewrite the equilibrium equations. We define

\[ \hat{m}_t = m_t \lambda_t^{\frac{1}{\bar{\sigma}}}, \hat{\varphi}_t = \varphi_t \lambda_t^{\frac{1}{\bar{\sigma}}}, \hat{e}_t = e_t \lambda_t^{\frac{1}{\bar{\sigma}}} \]  

(46)

\[ \hat{\Gamma}^\varphi_t = \Gamma^\varphi_t \lambda_t^{\frac{1}{\bar{\sigma}}}, \hat{\Gamma}^e_t = \Gamma^e_t \lambda_t^{\frac{1}{\bar{\sigma}}} \]  

(47)

and

\[ \hat{G} (R, h_t) = \hat{m}_t + \hat{\varphi}_t + \hat{e}_t + \hat{\Gamma}^\varphi_t + \hat{\Gamma}^e_t. \]  

(48)
Then the goods market equilibrium (12) can be rewritten as

$$h_t = \hat{G}(R, h_t) \frac{1}{w_l l_t^\gamma}.$$  \hspace{1cm} (49)

Time spent working $l_t$ can be written as a function of the growth rate of human capital using the equation (3). Using (48), (49), (3) and (39) we can characterize the entire equilibrium by a second order difference equation in human capital

$$\frac{h_{t+1}}{h_t} = \frac{\hat{G}(R, h_{t+1})}{\hat{G}(R, h_t)} \left( \frac{\phi - \frac{h_{t+1}}{h_t} + 1 - \delta}{\phi - \frac{h_{t+2}}{h_{t+1}} + 1 - \delta} \right) \left\{ \beta (\phi + 1 - \delta) \right\}^{\frac{1}{\gamma}}.$$  \hspace{1cm} (50)

Because we know the long run characteristics, we will use backward induction to solve the difference equation in human capital (50). We assume that for some high enough level of human capital, say $h_{T+1} \approx 10^{13}$, the economy is on the balanced growth path, $h_{T+1} = \lim_{t \to \infty} \left( \frac{h_{t+1}}{h_t} \right) = \left\{ \beta (\phi + 1 - \delta) \right\}^{\frac{1}{\gamma}}$. Then we apply the Newton-Raphson method to find $h_t$ knowing $h_{t+1}$ and $h_{t+2}$ for all $t$. In this way we obtain a numerical policy function $h_{t+1} = H(h_t; \alpha_e, \alpha_\phi, \beta, \delta, \phi, \theta, F, R, w)$. Given the initial level of human capital and the policy function, the behavior of all other variables in the economy can be calculated from the equations (29)-(38) and (46)-(47).

### 5.2 Calibration

We concentrate on the US and Norway between 1991 and 2007, the period in which we have the data available for both countries. We calibrate the model using yearly data. One period in the model corresponds to one year. As illustrated in section 2, the behavior of all payment instruments in the US is very regular between 1991 and 2001, but it changes later on. We thus assign common parameter values for the period 1991-2001, which will be our baseline calibration for the US. Then we identify variables responsible for changes after 2001 and change their values. As suggested above, the start of direct pricing of payment instruments in Norway in 1995 originates differences in trend in the usage of paper-based and electronic payment methods. The baseline set of parameters for Norway fits the period 1991-1995. We then change the value of the parameter responsible for the change in the price of the paper-based instrument.

The discount factor in the utility function is set to $\beta = 0.995$. The depreciation of human capital is set to $\delta = 0.025$ as in Gillman, Kejak, Valentinyi (1999). The values for the long run growth rates of the two economies correspond to the growth rate of output per worker found in the data, $g_{USA} = 1.021$ and $g_{NOR} = 1.029$. We set the efficiency of learning in the US to $\phi_{USA} = 0.055$ which implies that the inverse of the intertemporal elasticity of substitution is $\theta = 1.32$. We consider that the value of the elasticity of substitution is the same in both countries. As Norway displays higher growth rate, it means that the efficiency of learning is higher, $\phi_{NOR} = 0.069$. All these previously mentioned values imply that on the balanced growth path in both economies about 21% of time is devoted to working and the rest to studying. Average values of the 12 month effective rate imply that the baseline monetary policies are $R_{USA} = 1.05$ and $R_{NOR} = 1.07$, respectively. Marginal product of

\(^{11}\)For the calibration we use data from the FRED\textsuperscript{®} Economic Data, PennWorld Tables 6.3, World Development Indicators database and Norges Bank.
capital in goods production is set to $w = 1$. The parameters related to the intermediation function are set in order to fit the growth rates of cash, checks and cards in the data and reflect the usage of each payment instrument at the initial year: the barrier for electronic payments $F^{USA} = 1$ and $F^{NOR} = 2.5$, the technology diffusion coefficients $\alpha^{USA}_e = 1.47$ and $\alpha^{USA}_c = 1.8$, and $\alpha^{NOR}_e = 0.8$ and $\alpha^{NOR}_c = 1.2$. Table 1 summarizes the choice of the baseline parameter values for both countries. In order to calibrate the model to changes in agents’ payment choice we introduce several modifications. First, to account for the increase in the price of checks after 1995 in Norway we decrease step by step the technology diffusion into the paper-based payments, see the details on values of $\alpha^{NOR}_\varphi$ in each year in Table 2 (all other parameters remain unchanged at their baseline values). Second, to account for the changes in the usage of cash and checks in the US after 2001, we relax the monetary policy, we decrease $R$, as observed in the data, and we associate the drop in the usage of checks to an improvement in the relative position of electronic payments, a decrease in the barrier to electronic payments $F$.$^{12}$ This improvement of electronic payments against checks comes first from the fact that the Federal Reserve has undertaken measures to make electronic retail payments more attractive to consumers. In particular, the Federal Reserve has reduced prices for electronic payment transactions in order to promote this method of payment while still improving payment quality, Felsenfeld and Bilali (2005). Second, the events of September 11, 2001 broke the traditional characteristics of paper instruments underlined by Greenspan (2000): they allow the users themselves, within a structured format, to have significant control over when, where, and how to make payments. This disruption may have helped the decreasing tendency of check use observed since then. Table 2 again shows the values given to $R^{USA}$ and $F^{USA}$ for each year (as before, all other parameters are kept equal as in the baseline case).

<table>
<thead>
<tr>
<th></th>
<th>$g^*$</th>
<th>$R$</th>
<th>$\beta$</th>
<th>$w$</th>
<th>$\delta$</th>
<th>$\theta$</th>
<th>$\phi$</th>
<th>$l^*$</th>
<th>$F$</th>
<th>$\alpha_\varphi$</th>
<th>$\alpha_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>USA</td>
<td>1.021</td>
<td>1.05</td>
<td>0.995</td>
<td>1</td>
<td>0.025</td>
<td>1.32</td>
<td>0.055</td>
<td>0.21</td>
<td>1</td>
<td>1.47</td>
<td>1.8</td>
</tr>
<tr>
<td>NOR</td>
<td>1.029</td>
<td>1.07</td>
<td>0.995</td>
<td>1</td>
<td>0.025</td>
<td>1.32</td>
<td>0.069</td>
<td>0.21</td>
<td>2.5</td>
<td>0.8</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Table 1: Baseline calibration of the model for the US and Norway.

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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha^{NOR}_\varphi$</td>
<td>1.07</td>
<td>1.07</td>
<td>1.0</td>
<td>0.9</td>
<td>0.9</td>
<td>0.85</td>
<td>0.75</td>
<td>0.65</td>
<td>0.55</td>
<td>0.45</td>
<td>0.35</td>
<td>0.25</td>
</tr>
<tr>
<td>$R^{USA}$</td>
<td>1.05</td>
<td>1.05</td>
<td>1.05</td>
<td>1.05</td>
<td>1.05</td>
<td>1.0475</td>
<td>1.045</td>
<td>1.04</td>
<td>1.0375</td>
<td>1.035</td>
<td>1.035</td>
<td>1.035</td>
</tr>
<tr>
<td>$F^{USA}$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.9</td>
<td>0.85</td>
<td>0.8</td>
<td>0.8</td>
</tr>
</tbody>
</table>

Table 2: Calibration of the parameters responsible for the modification of the agent’s payment choice; $\alpha^{NOR}_\varphi$ – technology diffusion into the paper-based payments in Norway, $R^{USA}$ – nominal interest factor in the US, $F^{USA}$ – barrier to electronic era in the US.

$^{12}$A decrease in the usage of checks could be also explained via an increase of prices of checks, as for the case of Norway.
5.3 The dynamics of the payment instrument choice

We use the baseline calibration for the US to expose the model behavior. We simulate the evolution of the economy using the solution of the above mentioned second order difference equation (50). In Figure 5 we plot the relative usage of each instrument over a very long period of time. The initial value of human capital in that case is $h_1 = 1$. We begin at a very low stage of development during which only two means of payments coexist, cash and paper-based ones. With the development of new technologies paper-based payments become relatively cheaper and the fraction of markets where this payment method is chosen increases, meanwhile the fraction of markets where cash is employed decreases.

Figure 5: Evolution of the fraction of cash, paper-based and electronic payments as a percentage of all kinds of transactions for the baseline parameters for the US; behavior of the economy over 500 years, $h_1 = 1$.

Electronic payments emerge at a certain level of technology, in this particular case displayed in Figure 5, $h_{100} = 8$. Since that point agents choose between three payment instruments. With the development of new technologies the cost of electronic payments becomes relatively cheaper than the cost of other instruments, paper-based instruments and government money are driven away from the economy. However, the process of transformation of the payment system may take many years. In Figure 6 we plot the evolution of cash, paper-based and electronic purchases, as well as the intermediation payments for non-cash consumptions and the evolution of human capital. For $\alpha_e > 1$ and $\alpha_e - \alpha_p < 1$, cash and the intermediation cost of electronic payments grow at negative rates, and paper-based payments and the intermediation cost of paper-based purchases grow at positive rates, as shown in the Proposition 1. We can also see that the human capital growth rate is very stable over time. Figure 6 helps us to see how the growth rate of electronic consumption converges to the
one of human capital, and how the growth rate of cash consumption converges to the growth rate of intermediation payments for electronic transactions in the course of time (a result found analytically for the balanced growth path).

Figure 6: Evolution of the cash consumption $m_t$, paper-based consumption $\varphi_t$, electronic consumption $e_t$, payments to intermediary for both non-cash transactions, $\Gamma_t^\sigma$ and $\Gamma_t^e$, and human capital $h_t$, for the baseline parameters for the US.
5.3.1 Effects of the monetary policy

We analyze the effects of changes in the monetary policy. Let us assume that the opportunity cost of holding money increases. The 'cash line' in Figure 4 moves up. Agents will want to economize on their money holdings, substituting away from money. In the low stage of development they will substitute cash for paper-based payments, i.e. \( \frac{dx_t}{dt} > 0 \). On the higher level of development they will substitute cash for electronic payments, \( \frac{ds_t}{dt} > 0 \). Paper-based payments will be used on the same fraction of markets as under a low nominal interest rate, because the cutoff market \( \chi_t \) is not affected directly by the monetary policy, \( \frac{dx_t}{dt} = 0 \), see also the equation (31). The transformation of the payment methods under different monetary policies (and hence different inflation rates) is illustrated in Figure 7. In Figure 7a all arrows begin at the same level of human capital, \( h_{109} = 9.7 \) and show the transformation of the economy over 11 years.\(^{13}\) For a given level of technology, higher nominal interest rate implies less cash and more electronic transactions (the arrows for higher \( R \) begin always more on the left and higher). We can observe that the transformation of the payment system is slowing down with inflation (arrows are shorter). For a given initial condition, higher inflation rate means that the economy is closer to its balanced growth path.\(^{14}\)

![Figure 7](image-url)

(a) (b) (c)

Figure 7: Substitution of cash and paper-based payments by electronic ones under different monetary policies: \( R = 1.033, 1.05, 1.15 \) and 1.35 correspond to the inflation factor \( \pi = 1.0, 1.016, 1.113 \) and 1.31, respectively.

In Figure 7b we can see that since the moment in which the electronic payments arise, the share of paper-based payments is independent of the monetary policy, as the cutoff market \( \chi_t \) is independent

\(^{13}\) The point where the arrow begins for the baseline monetary policy, \( R = 1.05 \), has the properties of the payment behavior in the US in 1991.

\(^{14}\) It is a kind of convergence behavior. The larger is the initial distance of a country from its balanced growth path, the more rapid is the convergence towards it. The balanced growth path transformation of the payment system is represented by the left top corner where cash and paper-based payments are negligible.
of \( R \). The initial dependence of electronic consumption on \( R \) tends to disappear in the long run, as the fraction of markets where electronic transactions are employed increases to 1, \( \lim_{t \to \infty} s_t = 1 \), and the Lagrange multiplier converges to zero, see equation (36). Concerning the long run level of cash consumption, it is affected by the opportunity cost of holding money, as implied by the equation (33) and (25), see Figure 7c.

5.3.2 Effects of the growth rate (efficiency of studying)

![Figure 8: Behavior of the model under different balanced growth path growth rates (efficiency of studying), \( g^* = 1.021, 1.03, 1.04, 1.05 \) (\( \phi = 0.058, 0.07, 0.083 \) and 0.097).](image)

We make the efficiency of studying be responsible of changes in the growth rate of the economy. The growth rate of development of new technologies affects the speed of transformation of the payment system, see Figure 8a. Higher growth rate of human capital lowers faster the intermediation cost and the electronic payments era arises sooner, Figure 8b. The growth rate of electronic payments increases. The growth rate of other payment instruments depends on the values of the coefficients that characterize the diffusion of new technologies into the payment cost. For our calibration, the growth rate of cash decreases \( (1 - \alpha_e < 0) \) and the growth rate of paper-based payments increases \( (1 - \alpha_e + \alpha_p > 0) \), as can be observed in Figure 8c.
5.3.3 Effects of the barrier to the electronic payments era

Let us consider an increase in the fixed cost of electronic payments, for example a lower investment in infrastructure or more complicated geographical factors. The effect of an increase in $F$ can be again deduced from Figure 4. Moving the 'electronic payments line' upward we can see that $\frac{ds}{dF} < 0$ and $\frac{d\chi_t}{dF} > 0$. Electronic payments become relatively more expensive and it will be less profitable for agents to choose this instrument at lower level of development. The substitution of cash and paper-based transactions by electronic ones will begin later. The behavior of the economy for different levels of fixed cost is illustrated in Figure 9. We can see that the initial position of a country and the speed of transformation can be affected in an important way by changes in $F$, see the arrows in Figure 9a.\textsuperscript{15} When the barrier to electronic payments is higher, it will slow down the arrival of the electronic payments era and the process of substitution of cash by paper-based instruments during the low development lasts longer. Paper-based instruments are more widespread and we observe higher level and higher share of paper-based payments at the moment of the emergence of the electronic payments era. Once the electronic payments arise they begin to substitute both previously used instruments. We can see that changes in $F$ affect mostly the relative usage of electronic and paper based payments, not much the level of cash transactions, even if both cutoff indexes $s_t$ and $\chi_t$ depend on $F$.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure9.png}
\caption{Behavior of the model under different fixed costs in the intermediation function for electronic payments, $F = 0.5, 0.75, 1$ and 1.5.}
\end{figure}

\textsuperscript{15}Again, all arrows begin at the same level of human capital, $h_{109} = 9.7$, and show the changes in the payment choice decisions over 11 years.
5.3.4 Effects of the degree of technology diffusion into the electronic payments

The condition for the existence of electronic payments is $\alpha_e > \alpha_p$, as argued in the intermediation cost specification section. We maintain the baseline calibration for the US ($\alpha_p = 1.47$) and change $\alpha_e$. For a given level of development, a higher degree of technology diffusion into electronic payments makes the electronic transactions cheaper and more widespread across markets. It means that for a given level of technology the economy is closer to its balanced growth path and the transformation of the payment system is slower, see Figure 10a. The evolution of the share of each payment method over time depicted in Figure 10b is similar to the case of a decrease in $F$. The implications of an increase in the degree of technology diffusion into electronic payments are however very different when we look at the levels of each kind of consumption. The reason is that the value of $\alpha_e$ affects the long run growth rates of cash and paper-based transactions, Figure 10c. Lower price of electronic goods makes its usage more widespread and cash and paper-based payments are driven away faster.

![Figure 10: Behavior of the model under different degrees of technology diffusion into the electronic payments, $\alpha_e = 1.65$, 1.8, 2 and 2.5.](image)

Figure 10: Behavior of the model under different degrees of technology diffusion into the electronic payments, $\alpha_e = 1.65$, 1.8, 2 and 2.5.
5.3.5 Effects of the degree of technology diffusion into the paper-based payments

Higher degree of the technology diffusion into the paper-based payments makes checks cheaper relative to electronic cards. Under low development (era of the coexistence of cash and paper-based payments) an increase in $\alpha_\varphi$ implies much faster substitution of cash by checks. Once in the electronic payments era, an increase in $\alpha_\varphi$ causes an increase in the growth rate of paper-based transactions. Higher $\alpha_\varphi$ represents lower price of checks. When the price is lower, checks are employed in higher fraction of markets, decreasing the share of electronic payments. As can be seen in Figure 11, the effect of a variation in $\alpha_\varphi$ on cash payments is rather small. We can see from Figure 11a that all arrows begin and end on the same level of cash share in all payments, and they only differ in the share of electronic transaction as percentage of all non-cash transactions. As $\alpha_\varphi$ affects the growth rate of checks, we observe big differences in levels of paper-based consumption.\(^{16}\)

\[\text{Figure 11: Behavior of the model under different degrees of technology diffusion into the paper-based payments, } \alpha_\varphi = 0.5, 1, 1.47 \text{ and } 1.65.\]

\(^{16}\)This is the kind of behavior we observe in Norway. An increase in the price of checks drives out paper-based transactions out of the markets in favor of electronic payments.
5.4 Data versus Model

5.4.1 Comparison of the US data with the simulations

As mentioned previously, the behavior of the analyzed payment instruments is very regular in the US data for the interval between 1991-2001. The baseline calibration is applied to fit this particular interval. For the period 2001-2007 we have to introduce changes in the calibration of the model. We link the slowdown in the usage of electronic payments and an increase in the usage of cash after 2001 to lower interest rates. The decrease of relative usage of checks can be attributed to improvements in the electronic payments infrastructure or higher prices of checks. We perform the simulations assuming that the barrier to electronic payments decreases. Decreasing both the nominal interest rate and the barrier to electronic payments, we observe a shift from paper-based payments towards cash as observed in the data. Detail on changes of the parameters can be found in Table 2. In Figure 12a we plot the relationship between the share of cash transactions in all types of payments and the share of electronic transactions in non-cash payments in the model and the data. The arrows show the transformation that the payment system undergoes in 16 years in two steps, 1991-2001 and 2001-2007. We compare the evolution of the share of all three kinds of payment instruments in the model and the data in Figure 12b.

![Figure 12: Behavior of the payment methods in the model and the US data between 1991 and 2007.](image)

The results are essentially the same when we decrease the diffusion of technology into paper-based payments.
5.4.2 Comparison of the Norwegian data with the simulations

We present now the comparison of the model results with the data for Norway. A decrease in the degree of technology diffusion into paper-based transactions mimics the increase in prices of checks after 1995. We decrease $\alpha_p$ from 1.1 to 0.25 between the years 1996 and 2007, see details for each year in Table 2. The transformation of the payment system and the relative usage of each payment instrument is shown in Figure 13.

![Figure 13: Behavior of the payment methods in the model and the Norwegian data between 1991 and 2007.](image)

Figure 13: Behavior of the payment methods in the model and the Norwegian data between 1991 and 2007.

Results from Norway are quite interesting in terms of the structural change, and its speed, in consumer payment behavior. Let us recall at this point Figure 5 where the evolution of the economy over many periods is plotted. We can see that without changing any parameters it might take about 200 years to transform the payment system of an economy in a way that 75% of all payments are performed electronically. The example of Norway shows us that this can be achieved in less than 20 years by pricing correspondingly the most expensive instrument, checks. Implied welfare gains of such a policy are discussed in the next section.
6 Welfare

We want to evaluate welfare effects of changes in parameters that alter the usage of different payment instruments. As in the previous literature, a switch towards non-cash payments means that more resources are channeled towards financial intermediaries, and welfare losses arise. See for the case of two payment instruments for example Ireland (1994a), Marquis and Reffett (1994) or Hromcová (2008). Here we show that the introduction of electronic payments in addition to paper-based payments may have a positive effect on welfare.

The measure of welfare used is the one generally employed in the literature, as defined for example in Lucas (2003)

\[ U \left[ (1 + \varepsilon) c_A \right] = U \left( c_{REF} \right). \]

Number \( \varepsilon \), in units of a percentage of consumption goods, gives us welfare gain or loss of following a policy \( A \) with respect to a reference case \( REF \). We find \( \varepsilon \) that satisfies

\[
\sum_{t=1}^{\infty} \beta^t \left\{ \min \{ \bar{\chi}_t, \chi_t \} \frac{((1 + \varepsilon) c_{t}^{\varepsilon} (\cdot))^{1-\theta} - 1}{1 - \theta} + \max \{ \bar{\chi}_t, s_t \} - \min \{ \bar{\chi}_t, \chi_t \} \frac{((1 + \varepsilon) c_{t}^{\varepsilon} (\cdot))^{1-\theta} - 1}{1 - \theta} \right\} 
+ \sum_{t=1}^{\infty} \beta^t \left\{ \left[ 1 - \max \{ \bar{\chi}_t, s_t \} \right] \frac{((1 + \varepsilon) c_{t}^{\varepsilon} (\cdot))^{1-\theta} - 1}{1 - \theta} \right\} = \\
= \sum_{t=1}^{\infty} \beta^t \left\{ \min \{ \bar{\chi}_t^{REF}, \chi_t^{REF} \} \frac{c_{t}^{\varepsilon\, REF} (\cdot)^{1-\theta} - 1}{1 - \theta} \right\} 
+ \sum_{t=1}^{\infty} \beta^t \left\{ \max \{ \bar{\chi}_t^{REF}, s_t^{REF} \} - \min \{ \bar{\chi}_t^{REF}, \chi_t^{REF} \} \frac{c_{t}^{\varepsilon\, REF} (\cdot)^{1-\theta} - 1}{1 - \theta} \right\} 
+ \sum_{t=1}^{\infty} \beta^t \left\{ \left[ 1 - \max \{ \bar{\chi}_t^{REF}, s_t^{REF} \} \right] \frac{c_{0\, REF} (\cdot)^{1-\theta} - 1}{1 - \theta} \right\}.
\]

We can see from Figure 6 that the intermediation cost for electronic payments is much lower than the one for paper-based transactions. This means that a switch towards electronic payments reduces the resources which are unproductively channeled to intermediaries. That is translated into an increase in welfare.

In the first step of our analysis we study the welfare effects of advancing or delaying of the arrival of the electronic era. The reference case is the baseline calibration for the US. We change each of the parameters in the intermediation cost and the monetary policy in a way that the electronic payments are employed 20, 10 and 5 years earlier (later) than in the reference case. The results are summarized in Table 3. The initial value for all cases is \( h_1 = 8 \). This initial value implies that in the reference case the electronic era just begins at \( t = 1 \). Welfare is evaluated over 1000 periods. Friedman rule is the welfare maximizing policy. Nevertheless, \( R = 1 \) implies an economy where only cash payments are used and no transformation of the payment system occurs, which obviously does not coincide with the observed empirical facts. For example, according to this exercise, making the paper-based payments more expensive (cheaper) in a way that agents start to use the electronic transactions 5 years earlier (later) implies welfare gain (loss) of 4.3% (5.4%) of the stream of consumption. The same changes in
the behavior of agents induced by an analogous change in the price of electronic transactions imply
2.3% gain and 2.8% loss, respectively. Changes due to different barriers to electronic payments cause
similar welfare effects. The behavior of the model is different when concerning the changes in the
opportunity cost of holding money. A higher interest rate implies that electronic payments are used
earlier, but as more resources go to intermediaries, this leads to welfare losses. In Table 3 we can
see that an increase (decrease) in inflation that makes agents use the electronic payments 5 years
earlier (later) implies a welfare loss (gain) of 0.2% of the stream of consumption. We can see that
an increase in prices of checks due to direct pricing policy makes the economy better off in a very
favorable way.

In the second step of our welfare analysis we calculate the welfare gain implied by our model for
the changes in prices of paper-based instruments experienced by Norway. We take as a reference
the Norwegian economy under the calibration that fits the period 1991-1995. We assume that these
parameters do not change over time and we evaluate the welfare obtained. Then we evaluate the
welfare under the calibration that fits the whole interval analyzed, 1991-2007. We find that the
welfare gain reaches 4% of the stream of consumption.

<table>
<thead>
<tr>
<th></th>
<th>Friedman rule</th>
<th>−20</th>
<th>−10</th>
<th>−5</th>
<th>REF</th>
<th>+5</th>
<th>+10</th>
<th>+20</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F$%</td>
<td>Friedman rule</td>
<td>0.4</td>
<td>0.64</td>
<td>0.80</td>
<td>1.00</td>
<td>1.25</td>
<td>1.55</td>
<td>2.35</td>
</tr>
<tr>
<td>$\varepsilon$%</td>
<td></td>
<td>−8.0</td>
<td>−5.6</td>
<td>−3.5</td>
<td>−2.0</td>
<td>–</td>
<td>2.5</td>
<td>5.2</td>
</tr>
<tr>
<td>$\alpha_e$%</td>
<td>Friedman rule</td>
<td>2.1</td>
<td>1.92</td>
<td>1.86</td>
<td>1.80</td>
<td>1.75</td>
<td>1.71</td>
<td>1.64</td>
</tr>
<tr>
<td>$\varepsilon$%</td>
<td></td>
<td>−8.0</td>
<td>−6.1</td>
<td>−3.8</td>
<td>−2.3</td>
<td>–</td>
<td>2.8</td>
<td>5.9</td>
</tr>
<tr>
<td>$\alpha_\varphi$%</td>
<td>Friedman rule</td>
<td>0*</td>
<td>1.10</td>
<td>1.33</td>
<td>1.47</td>
<td>1.55</td>
<td>1.62</td>
<td>1.69</td>
</tr>
<tr>
<td>$\varepsilon$%</td>
<td></td>
<td>−8.0</td>
<td>−7.2</td>
<td>−6.4</td>
<td>−4.3</td>
<td>–</td>
<td>5.4</td>
<td>14.2</td>
</tr>
<tr>
<td>$R(\pi)$</td>
<td></td>
<td>1.00 (0.968)</td>
<td>1.130 (1.094)</td>
<td>1.080 (1.045)</td>
<td>1.0630 (1.029)</td>
<td>1.050 (1.016)</td>
<td>1.040 (1.007)</td>
<td>1.032 (1.00)</td>
</tr>
<tr>
<td>$\varepsilon$%</td>
<td></td>
<td>−8.0</td>
<td>0.7</td>
<td>0.3</td>
<td>0.2</td>
<td>–</td>
<td>−0.2</td>
<td>−0.7</td>
</tr>
</tbody>
</table>

Table 3: Welfare loss (positive)/gain (negative), for different parameters that affect the choice of payment
methods with respect to the baseline calibration of the US economy: $\varepsilon$—percentage of the stream of
consumption to be added (positive) or taken (negative) in order to obtain the same welfare as under the
reference policy $REF$, $R$—nominal interest factor, $\pi$—inflation factor, $F$—barrier to electronic payments,
$\alpha_e$—degree of knowledge diffusion into electronic payments, $\alpha_\varphi$—degree of knowledge diffusion into paper-
based payments. The initial value corresponds to $h_1=8$; in the first row −20, −10 and −5 (20, 10, and 5)
mean that electronic payments are started to be used 20, 10 or 5 years earlier (later) than in the reference
case $REF$.

* This value advances the emergence of the electronic era only 17 periods. It is not possible to advance it more periods changing this parameter.
7 Conclusions

We present a general equilibrium model on payment choice at a retail level which provides theoretical foundations to reconcile previous empirical evidence on payment choice among cash, electronic and paper-based instruments. Consumer choice of a particular payment instrument depends on parameters like the payment infrastructure, the cost of each payment method, the growth rate of the economy, the degree of technology development or the interest rate. These variables help to explain why countries with similar payment alternatives end up with very different usage of alternative payment methods.

Due to the development of new technologies financial intermediation is cheaper and new payment methods may emerge endogenously at a certain degree of development. In our model agents use initially cash and paper-based payments, and at some point in time they start using also electronic methods. Electronic payments drive paper-based checks and cash away from the economy and transform the country payment system. The electronic era arrives earlier if the level of technology, growth rate of the economy, inflation rate, technology diffusion into the electronic payments are higher, if the payment infrastructure is more developed, or if the technology diffusion into the paper-based payments is lower (checks are more expensive). Similar effects speed up the transformation of the payment system.

The welfare analysis provides some interesting insights for the design and management of a payment system. Results show that the introduction of a cheaper payment instrument may induce important welfare improvement. The calibration exercise for Norway shows that the policy of direct pricing of payment instruments carried out by individual banks and encouraged by the Norwegian authorities may imply 4% increase in the welfare of the country. These results indicate that authorities should pursue policies of direct pricing of payment instruments according to their relative costs. Such policies could diminish the resources allocated to financial intermediaries and lead to an increase of the well-being of individuals.
References


